Control of industrial robots

Kinematic calibration

Prof. Paolo Rocco (paolo.rocco@polimi.it)

Politecnico di Milano, Dipartimento di Elettronica, Informazione e Bioingegneria

Kinematic calibration

- The Denavit-Hartenberg (DH) parameters of a robot need to be known as precisely as possible in order to increase the accuracy of the manipulator

- In general however they cannot be directly measured

- **Kinematic calibration** is a process, based on a series of measurements of the manipulator's end effector, that allows to obtain accurate estimates of the DH parameters

Direct kinematics

Consider a n d.o.f. manipulator, whose DH parameters are gathered in the following vectors:

$$\boldsymbol{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \qquad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \qquad \boldsymbol{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

The **direct kinematics** equation can be written as:

$$\boldsymbol{x} = \boldsymbol{k}(\boldsymbol{a}, \boldsymbol{\alpha}, \boldsymbol{d}, \boldsymbol{\theta})$$

where $\boldsymbol{x}$ are 6 operational space coordinates (position and orientation of the end-effector).

If the joints are rotational, θ_i are typically measured by encoders.

Why kinematic calibration

Kinematic calibration of a robot is needed for several reasons:

- **Tolerances** in mechanical building of components and in the assembly of the links and joints imply some errors
- **Encoder mounting** may not be consistent with the zero of the θ_i variables
- The kinematic structure of robot manipulators is in open chain and this tends to amplify the errors

The goal of the kinematic calibration is then to use an **external and accurate measurement device** in order to correct the nominal values of the DH parameters and then to improve the accuracy of the end effector pose.

Kinematic calibration has to be performed **only once** for each robot, before operation.

An example of calibration system

- A plate with spherical reflectors is mounted at the end effector
- A high precision vision system based on triangulation tracks such reflectors
- The end-effector pose is reconstructed

Linearizing the direct kinematics

Define:

$\boldsymbol{x}_{nom}$ the nominal pose that can be computed using the nominal values of the DH parameters
$\boldsymbol{x}_{act}$ the actual pose as measured by the vision system.

(nominal values for the joint variables are those returned by the transducers in the current posture).

Under the assumption of small deviations of the parameters, we can **linearize the direct kinematics**:

$$\Delta\boldsymbol{x} = \boldsymbol{x}_{act} - \boldsymbol{x}_{nom} = \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{a}}\Delta\boldsymbol{a} + \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{\alpha}}\Delta\boldsymbol{\alpha} + \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{d}}\Delta\boldsymbol{d} + \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{\theta}}\Delta\boldsymbol{\theta}$$

where the Jacobians of $\boldsymbol{k}$ with respect to $\boldsymbol{a}, \boldsymbol{\alpha}, \boldsymbol{d}$ and $\boldsymbol{\theta}$ are computed using the nominal DH parameters and $\Delta\boldsymbol{a}, \Delta\boldsymbol{\alpha}, \Delta\boldsymbol{d}$ and $\Delta\boldsymbol{\theta}$ are the deviations of the DH parameters with respect to their nominal values.

Linearizing the direct kinematics

Define the following vector and matrix:

$$\Delta\boldsymbol{\zeta} = \begin{bmatrix} \Delta\boldsymbol{a} \\ \Delta\boldsymbol{\alpha} \\ \Delta\boldsymbol{d} \\ \Delta\boldsymbol{\theta} \end{bmatrix}$$

$(4n \times 1)$ vector representing the deviations of the DH parameters

$$\boldsymbol{\Phi} = \begin{bmatrix} \dfrac{\partial \boldsymbol{k}}{\partial \boldsymbol{a}} & \dfrac{\partial \boldsymbol{k}}{\partial \boldsymbol{\alpha}} & \dfrac{\partial \boldsymbol{k}}{\partial \boldsymbol{d}} & \dfrac{\partial \boldsymbol{k}}{\partial \boldsymbol{\theta}} \end{bmatrix}$$

$(6 \times 4n)$ matrix, called **kinematic calibration matrix**

We can rewrite the equation of the linearized direct kinematics as:

$$\Delta\boldsymbol{x} = \boldsymbol{\Phi} \cdot \Delta\boldsymbol{\zeta}$$

How to compute $\Delta\boldsymbol{\zeta}$ given $\boldsymbol{x}_{nom}$, $\boldsymbol{x}_{act}$ and the matrix $\boldsymbol{\Phi}$ computed with the nominal parameters?

Calibration experiments

$$\Delta \boldsymbol{x} = \boldsymbol{\Phi} \cdot \Delta \boldsymbol{\zeta}$$

It is a system of 6 equations in $4n$ unknowns.

We need to perform a certain number l of experiments, each time changing the pose of the end-effector. Stacking the above equations referred to the various poses, we have:

$$\Delta \bar{\boldsymbol{x}} = \begin{bmatrix} \Delta \boldsymbol{x}_1 \\ \vdots \\ \Delta \boldsymbol{x}_l \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_1 \\ \vdots \\ \boldsymbol{\Phi}_l \end{bmatrix} \Delta \boldsymbol{\zeta} = \bar{\boldsymbol{\Phi}} \cdot \Delta \boldsymbol{\zeta}$$

calibration equation

computed with the nominal parameters

To avoid ill-conditioning of matrix $\bar{\boldsymbol{\Phi}}$, it is advisable to take a large number of poses so that $6l \gg 4n$

Calibration experiments

$\Delta\bar{\boldsymbol{x}} = \bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta}$

Matrix $\bar{\boldsymbol{\Phi}}$ has many more ($6l$) rows than columns ($4n$).
The system in the unknowns $\Delta\boldsymbol{\zeta}$ is **overdetermined**.

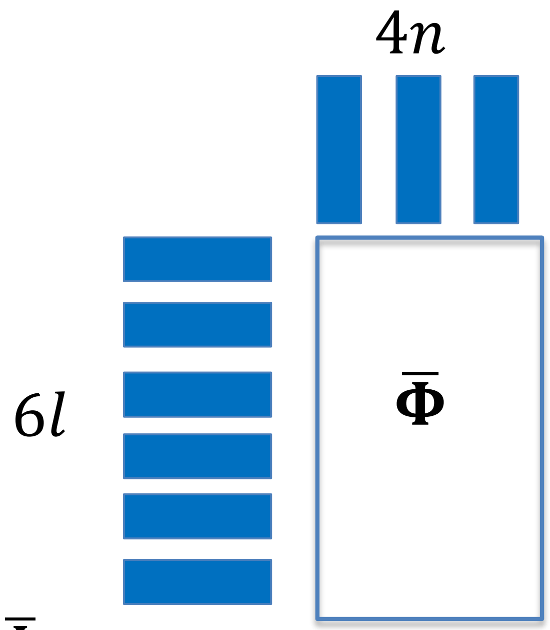
It can be solved in a **least-squares sense**:

$\Delta\boldsymbol{\zeta} = \bar{\boldsymbol{\Phi}}^{\#} \cdot \Delta\bar{\boldsymbol{x}}$

where $\bar{\boldsymbol{\Phi}}^{\#} = (\bar{\boldsymbol{\Phi}}^T\bar{\boldsymbol{\Phi}})^{-1}\bar{\boldsymbol{\Phi}}^T$ is the **left pseudoinverse** matrix of $\bar{\boldsymbol{\Phi}}$.
Notice that $\bar{\boldsymbol{\Phi}}^{\#}\bar{\boldsymbol{\Phi}} = \boldsymbol{I}$

This solution minimizes the quantity:

$||\bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta} - \Delta\bar{\boldsymbol{x}}||^2$



On the least-squares method

A short derivation of the result of the least-squares method is reported here.

Equation:

$$\Delta\bar{\boldsymbol{x}} = \bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta}$$

is solved for $\Delta\boldsymbol{\zeta}$ by minimizing the quantity:

$$\|\bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta} - \Delta\bar{\boldsymbol{x}}\|^2 = (\boldsymbol{\Phi}_1\Delta\boldsymbol{\zeta} - \Delta\boldsymbol{x}_1)^2 + \cdots + (\boldsymbol{\Phi}_l\Delta\boldsymbol{\zeta} - \Delta\boldsymbol{x}_l)^2$$

Taking the derivatives with respect to $\Delta\boldsymbol{\zeta}$ and equating them to zero:

$$(\boldsymbol{\Phi}_1\Delta\boldsymbol{\zeta} - \Delta\boldsymbol{x}_1)^T\boldsymbol{\Phi}_1 + \cdots + (\boldsymbol{\Phi}_l\Delta\boldsymbol{\zeta} - \Delta\boldsymbol{x}_l)^T\boldsymbol{\Phi}_l = \boldsymbol{0}$$

or:

$$(\bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta} - \Delta\bar{\boldsymbol{x}})^T\bar{\boldsymbol{\Phi}} = \boldsymbol{0}$$

Equivalently:

$$\bar{\boldsymbol{\Phi}}^T(\bar{\boldsymbol{\Phi}} \cdot \Delta\boldsymbol{\zeta} - \Delta\bar{\boldsymbol{x}}) = \bar{\boldsymbol{\Phi}}^T\bar{\boldsymbol{\Phi}}\Delta\boldsymbol{\zeta} - \bar{\boldsymbol{\Phi}}^T\Delta\bar{\boldsymbol{x}} = \boldsymbol{0}$$

Solving for $\Delta\boldsymbol{\zeta}$:

$$\Delta\boldsymbol{\zeta} = (\bar{\boldsymbol{\Phi}}^T\bar{\boldsymbol{\Phi}})^{-1}\bar{\boldsymbol{\Phi}}^T\Delta\bar{\boldsymbol{x}} = \bar{\boldsymbol{\Phi}}^{\#} \cdot \Delta\bar{\boldsymbol{x}}$$

Iterations

A first estimate of the DH parameters can then be obtained as:

$$\boldsymbol{\zeta}' = \boldsymbol{\zeta}_{\text{nom}} + \Delta\boldsymbol{\zeta}$$

Since the problem is nonlinear, the procedure can be **iterated** until $\Delta\boldsymbol{\zeta}$ converges under a certain threshold:

$$\Delta\bar{\boldsymbol{x}}' = \bar{\boldsymbol{\Phi}}' \cdot \Delta\boldsymbol{\zeta}$$

computed with the new values $\boldsymbol{\zeta}'$

The result of this process are more accurate estimates of the geometrical parameters and possible corrections to be made on the joint transducers.

With a good calibration of the geometric parameters, the **improvement** in the accuracy of the robot can increase by a **factor of 10.**

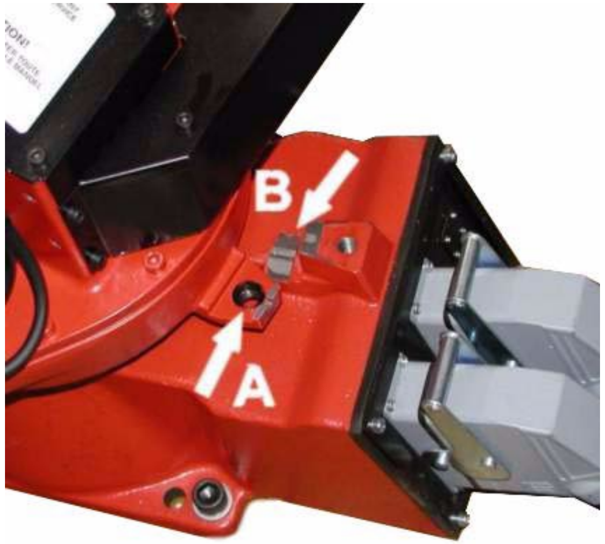
Calibration experiment

Link to the video: https://www.youtube.com/watch?v=WmLVzNjJZVs

Calibration of the transducers

The kinematic calibration described so far is performed by the robot manufacturer before the robot is in operation.

A different calibration, not to be confused with the kinematic one, can be made by the user at the startup of the robot and aims at guaranteeing that the **position transducer data** are consistent with actual positions of the joints. It can be performed with a dedicated instrument or just visually.



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