Control of Industrial Robots

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June 15, 2026

NAME:

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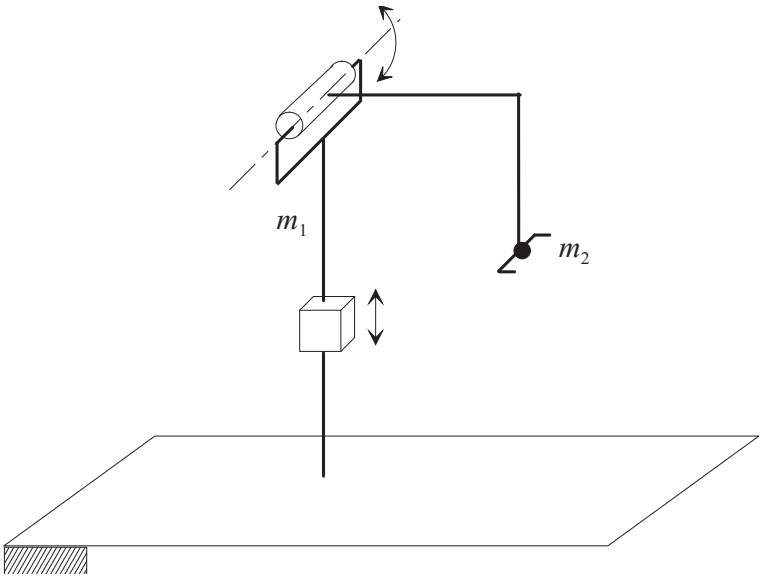
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the vector of the gravitational terms for this manipulator.

3. Write the expression of the PD + gravity compensation control law for this manipulator.

4. What is the theoretical property guaranteed by a PD + gravity compensation control scheme?

EXERCISE 2

1. Explain why for the kinematic scaling of trajectories it is possible to consider each joint separately, while for the dynamic scaling this is not possible.

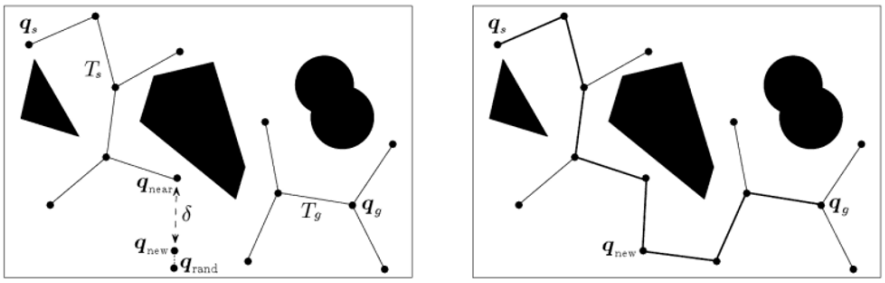
2. Consider the following equation:

$$\tau_i(t) = \alpha_i\left(r(t)\right)\ddot{r}(t) + \beta_i\left(r(t)\right)\dot{r}^2(t) + \gamma_i\left(r(t)\right)$$

   Explain whether such equation is used in the kinematic scaling or in the dynamic scaling and define all symbols used in the equation.

3. Assume now that, in a robot that is not affected by gravity, trajectories have been planned such that the torque of one joint exceeds its limit by 44% (the torques of the other joints are within their limits). Explain how the trajectory can be scaled and what should be the scaling factor.

4. With reference to the following picture, discuss the RRT method for the path planning with obstacle avoidance.



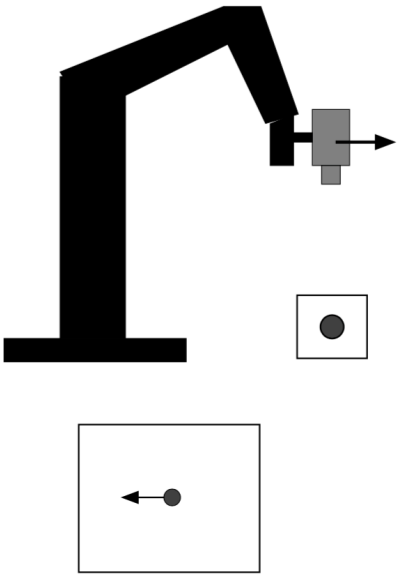
EXERCISE 3

Consider the control of a manipulator with vision sensors.

1. Explain what are the "eye-in-hand" and the "eye-to-hand" configurations, mentioning some pros and cons of both solutions.

2. Making reference to the following picture, where a single image point is considered, explain what is the interaction matrix in the context of visual control, specifying precisely:

   - the variables that are related by the interaction matrix
   - the size of the interaction matrix
   - the variables upon which the interaction matrix depends
   - which columns of the matrix depend on the depth Z



3. Discuss the concept of the null space of an interaction matrix, providing physical insight to the concept.

4. Explain what is the image Jacobian, what is the size of such matrix, and what is its relation with the interaction matrix.