Control of Industrial Robots

Prof. Rocco

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NAME:

UNIVERSITY ID NUMBER:

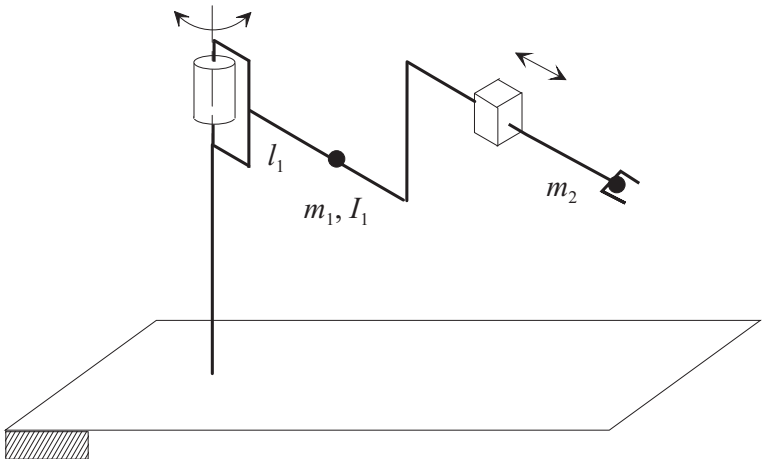
SIGNATURE: _____

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the complete dynamic model for this manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Write the equations of an inverse dynamics controller for this robot. What are the dynamic parameters that need to be identified in order to implement such controller?

EXERCISE 2

1. Explain what is the difference between the kinematic and the dynamic scaling of a trajectory

2. The parametric form of a cycloidal trajectory for kinematic scaling is given by:

$$\sigma(\tau) = \tau - \frac{1}{2\pi}\sin(2\pi\tau)$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time T and the total displacement h.

3. Consider the design of a cycloidal trajectory from $q_i = 0$ to $q_f = 20$, with $\dot{q}_{\max} = 20$ and $\ddot{q}_{\max} = 40$. Find the minimum positioning time.

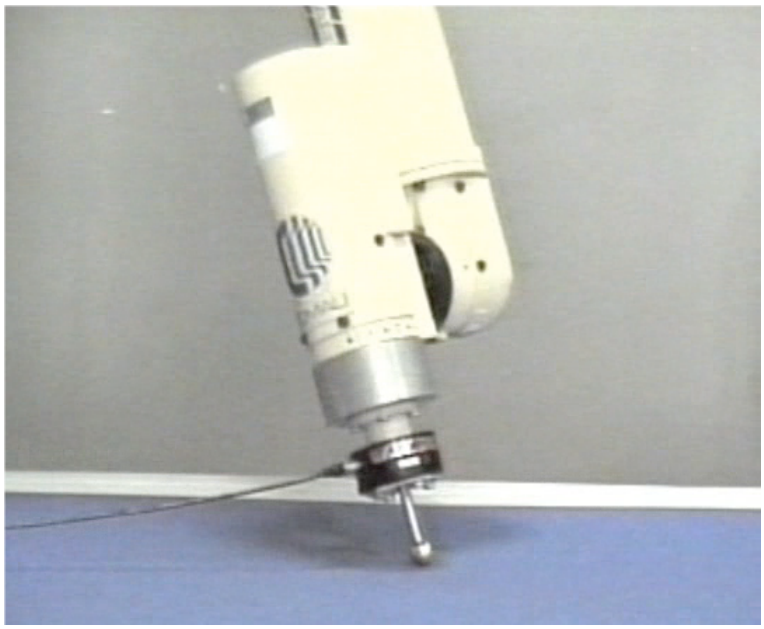
4. The result of a process of dynamic scaling is the following equation:

$$\tau_i(\theta) - g_i(\theta) = \frac{1}{k^2} \left[\tau_i(t) - g_i(t)\right]$$

Comment such equation and explain the meaning of symbols θ and k, as well as their relation.

EXERCISE 3

Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:



1. Assume a point contact and draw a contact frame directly on the picture. Based on this frame and neglecting angular velocities and moments, express the natural and the artificial constraints for this problem.

2. Write the expression of the selection matrix for this problem, explaining the meaning of such matrix.

3. Sketch the block diagram of a hybrid force/position controller. What happens if there is friction at the contact between the tool and the surface?

4. Suppose now that along the force controlled direction an explicit force controller has to be designed. Determine the expression of such controller, taking a bandwidth of 30 rad/s.