

Control of Industrial Robots

PROF. ROCCO

JULY 23, 2025

NAME:

UNIVERSITY ID NUMBER:

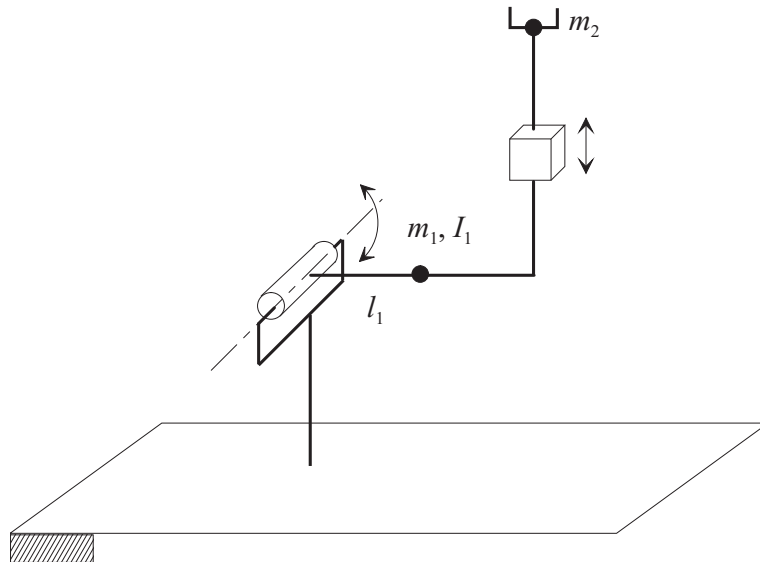
SIGNATURE: _____

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms¹ for this manipulator.

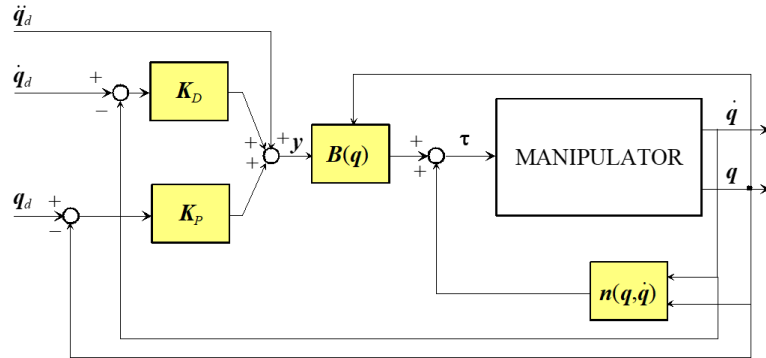
3. Ignoring the gravitational effects, write the dynamic model for this manipulator.

4. Write the dynamic model of this exercise in a form that is linear with respect to a set of dynamic parameters. In order to experimentally identify such parameters, which ones out of the position, velocity and acceleration of the two joint variables are needed?

¹The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2} \left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

EXERCISE 2

1. Consider the block diagram sketched in the following picture:



Explain which control scheme it refers to and what is the result in terms of closed-loop dynamics that can be achieved with such control scheme.

2. Consider now a generic two-link manipulator, for which the inertia matrix, the matrix of the Coriolis and centrifugal terms, and the vector of the gravitational terms can be written as follows, respectively:

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Write the expression (equation by equation) of the control law for the control scheme of this exercise, for this two d.o.f. manipulator.

3. Tune the two matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ in such a way that the dynamics of the error in the two joints is identical, with two real eigenvalues at frequencies 20 rad/s and 30 rad/s.
4. Consider now the version of the control scheme of this exercise in the operational space. Write the related control law and explain what result can be obtained in this case.

EXERCISE 3

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta \mathbf{x} = \mathbf{\Phi} \Delta \zeta$$

Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

3. Based on the equation previously commented, explain how the kinematic calibration algorithm works.
4. Discuss the concepts of repeatability and accuracy of the robot. With a good kinematic calibration do you expect to improve the repeatability or the accuracy?