Control of Industrial Robots

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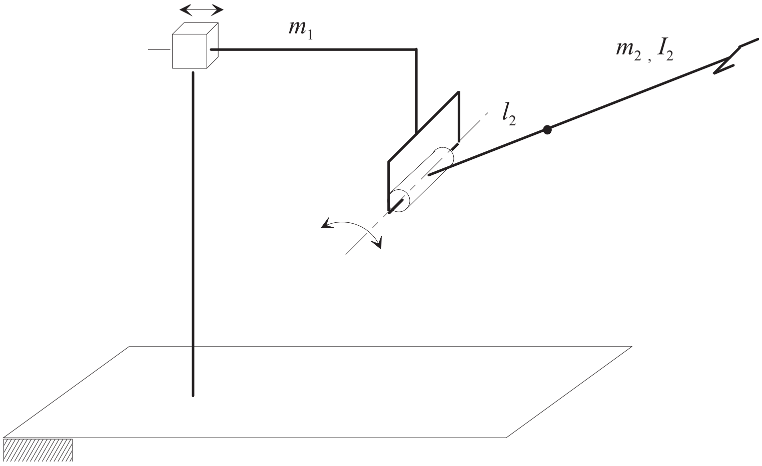
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1]

[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Ignoring the gravitational terms, write the equations of the dynamic model for this manipulator.

EXERCISE 2

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$
\begin{array}{ccccc}
t_1 = 0 & t_2 = 3 & t_3 = 5 & t_4 = 7 & t_5 = 10 \\
q_1 = 0 & q_2 = 45 & q_3 = 20 & q_4 = 50 & q_5 = 65
\end{array}
$$

   Consider the interpolation of such points by a single polynomial of suitable degree. Write the vector equation that has to be solved for this specific example in order to find the coefficients of such polynomial.

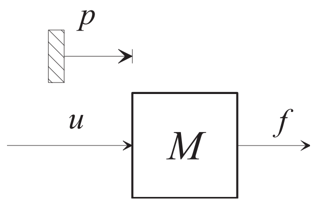
2. What are the issues that suggest not to use the interpolation with a single polynomial?

3. Assume now that you want to use cubic polynomials in each interval. Assign suitable values to the velocity at the intermediate points.

4. If you use the spline method to interpolate the given points, which ones out of the position, the velocity and acceleration are continuous in all the intermediate time instants, i.e in the open interval (t_1, t_5)? And what about the initial and the final time instants t_1 and t_5?

EXERCISE 3

1. Consider a single mass affected by an external force f and a control force u:



   Show that applying a proportional-derivative action on the position p yields an impedance relation. Are all the parameters of such relation assignable?

2. Write the expression of an impedance control law such that all the parameters of the impedance law can be assigned.

3. The generalization of the impedance control law of the previous question to a whole manipulator consists of the following equations:

$$
\begin{aligned}
\tau &= \mathbf{B}(\mathbf{q})\,\mathbf{y} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{J}^T(\mathbf{q})\,\mathbf{h} \\
\mathbf{y} &= \mathbf{J}_A^{-1}(\mathbf{q})\,\mathbf{M}_d^{-1}\left(\mathbf{M}_d\ddot{\mathbf{x}}_{\mathbf{d}} + \mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}} - \mathbf{M}_d\dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}})\,\dot{\mathbf{q}} - \mathbf{h}_A\right)
\end{aligned}
$$

Explain the meaning of all the symbols used in such equation.

4. Write the resulting impedance relation that is obtained using the control law of the previous question. If $\mathbf{M}_d, \mathbf{D}_d, \mathbf{K}_d$ are all diagonal matrices, what do you expect will happen if you apply a force at the end effector along a certain direction (for example along the axis $\mathbf{z}$)?