Control of Industrial Robots

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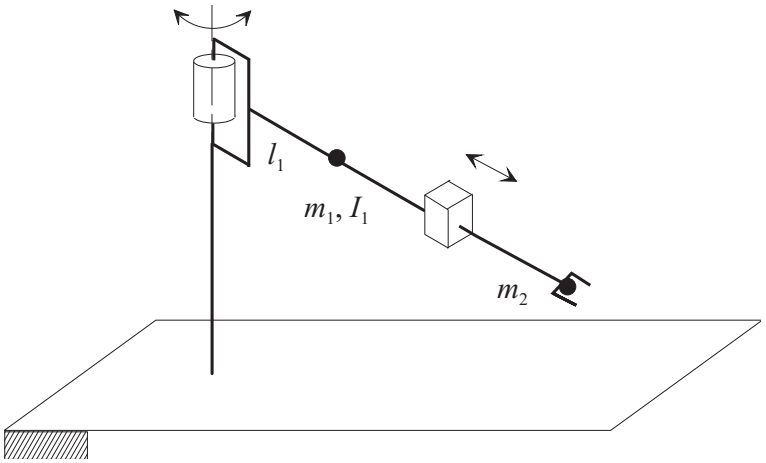
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator. Is this the only possible expression that matrix $\mathbf{C}$ can take?

3. Check that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator without gravity load, as the robot in this exercise, compute the expression of the derivative of the kinetic energy, exploiting the fact that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric. Specify whether this result can be obtained only with a matrix $\mathbf{C}$ computed from the Christoffel symbols or if it is general.

EXERCISE 2

Consider a kinematically redundant manipulator.

1. Express the solution of the inverse kinematics in the form that includes a closed loop correction (kinematic control) and explain why this correction is used.

2. Write the expression of the dynamics of the error between desired task variables and the task variables computed by the algorithm. What are the requirements on the gain of the algorithm for these dynamics to be asymptotically stable?

3. Consider now the motion of the end effector along a linear path. Assigning to the natural coordinate s a cubic dependence on time, derive the expressions of the maximum speed and the maximum acceleration as functions of the displacement h and the positioning time T.

4. Assume that the length of the segment to cover is $2m$, the maximum linear velocity of the end effector is $2m/s$ and the maximum linear acceleration $3m/s^2$. Compute the minimum positioning time, adopting a cubic dependence on time.

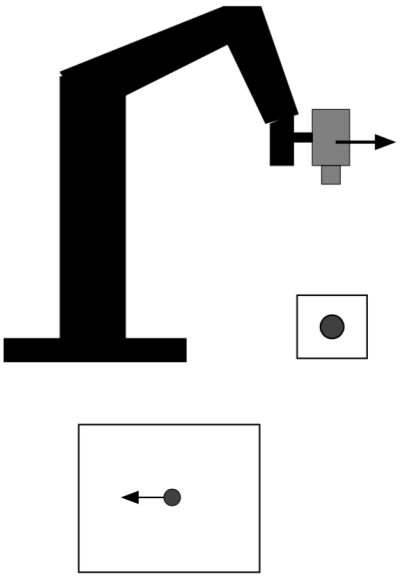
EXERCISE 3

Consider the control of a manipulator with vision sensors.

1. Explain what are the "eye-in-hand" and the "eye-to-hand" configurations, mentioning some pros and cons of both solutions.

2. Making reference to the following picture, where a single image point is considered, explain what is the interaction matrix in the context of visual control, specifying precisely:

   - the variables that are related by the interaction matrix
   - the size of the interaction matrix
   - the variables upon which the interaction matrix depends
   - which columns of the matrix depend on the depth Z



3. Explain what is the image Jacobian, what is the size of such matrix, and what is its relation with the interaction matrix.

4. Sketch the block diagram of a look-and-move, image-based, vision control system and specify the expression of a control law based on the image Jacobian.