Control of Industrial Robots

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SOLUTION

Control of Industrial Robots
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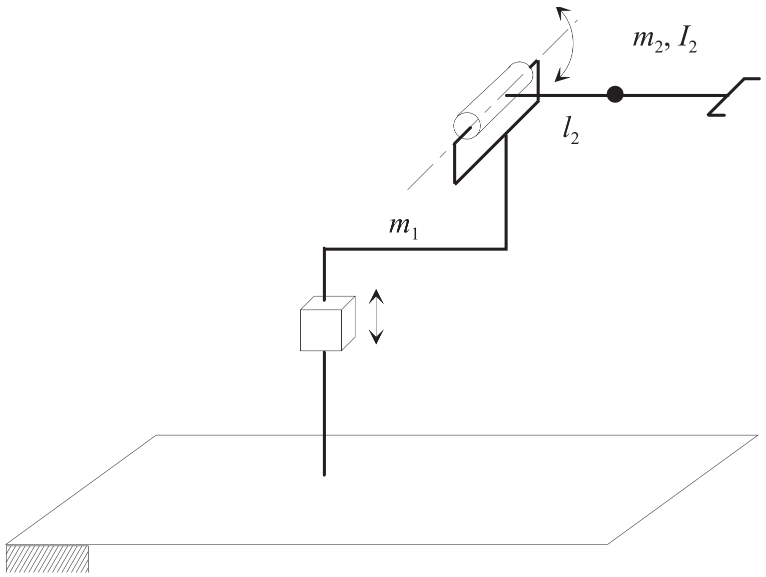
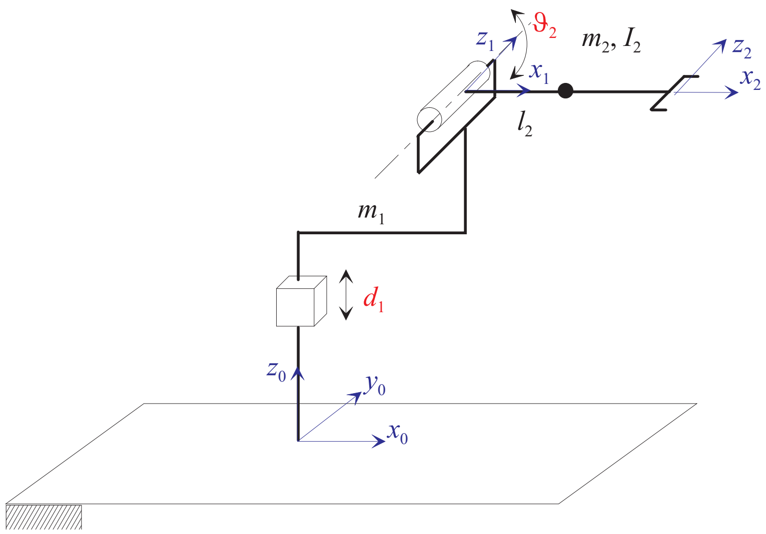
EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

Denavit-Hartenberg frames can be defined as sketched in this picture:



[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

Computations of the Jacobians:

Link 1

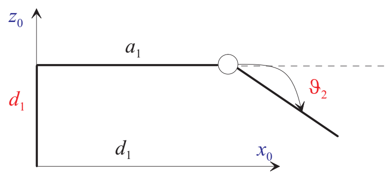
$$\mathbf{J}_P^{(l_1)} = \begin{bmatrix} \mathbf{j}_{P_1}^{(l_1)} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_0 & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Link 2

$$\mathbf{J}_P^{(l_2)} = \begin{bmatrix} \mathbf{j}_{P_1}^{(l_2)} & \mathbf{j}_{P_2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_0 & \mathbf{z}_1 \times (\mathbf{p}_{l_2} - \mathbf{p}_1) \end{bmatrix} = \begin{bmatrix} 0 & -l_2 s_2 \\ 0 & 0 \\ 1 & -l_2 c_2 \end{bmatrix}$$

$$\mathbf{J}_O^{(l_2)} = \begin{bmatrix} \mathbf{j}_{O_1}^{(l_2)} & \mathbf{j}_{O_2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{z}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

For the above computations, we can make reference to the following picture:



and to the following auxiliary vectors:

$$\mathbf{p}_{l_2} = \begin{bmatrix} a_1 + l_2 c_2 \\ 0 \\ d_1 - l_2 s_2 \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} a_1 \\ 0 \\ d_1 \end{bmatrix}, \mathbf{z}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

The inertia matrix can be computed now:

$$\begin{aligned} \mathbf{B}(\mathbf{q}) &= m_1 \mathbf{J}_P^{(l_1)T} \mathbf{J}_P^{(l_1)} + m_2 \mathbf{J}_P^{(l_2)T} \mathbf{J}_P^{(l_2)} + I_2 \mathbf{J}_O^{(l_2)T} \mathbf{J}_O^{(l_2)} \\ &= m_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} 1 & -l_2 c_2 \\ -l_2 c_2 & l_2^2 \end{bmatrix} + I_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \end{aligned}$$

where:

$$\begin{aligned} b_{11} &= m_1 + m_2 \\ b_{12} &= -m_2 l_2 c_2 \\ b_{22} &= I_2 + m_2 l_2^2 \end{aligned}$$

2. Compute the gravitational terms for this robot.

Since the vertical axis is the $\mathbf{z}_0$ axis pointing upwards, the gravity acceleration vector is:

$$\mathbf{g}_0 = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$

The gravitational torques are thus:

$$\mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix},$$

where:

$$g_1 = -m_1 \mathbf{g}_0^T \mathbf{j}_{P_1}^{(l_1)} - m_2 \mathbf{g}_0^T \mathbf{j}_{P_1}^{(l_2)} = -m_1 \mathbf{g}_0^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - m_2 \mathbf{g}_0^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = (m_1 + m_2)\, g$$

$$g_2 = -m_1 \mathbf{g}_0^T \mathbf{j}_{P_2}^{(l_1)} - m_2 \mathbf{g}_0^T \mathbf{j}_{P_2}^{(l_2)} = -m_2 \mathbf{g}_0^T \begin{bmatrix} -l_2 s_2 \\ 0 \\ -l_2 c_2 \end{bmatrix} = -m_2 g l_2 c_2$$

3. Ignoring the Coriolis and centrifugal terms, write the dynamic model of the manipulator and show that this model is linear with respect to a certain set of dynamic parameters.

   Neglecting Coriolis and centrifugal terms, the dynamic model can be written as:

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau$$

   The two equations that form the model are:

$$\begin{aligned} (m_1 + m_2)\, \ddot{d}_1 - m_2 l_2 c_2 \ddot{\vartheta}_2 + (m_1 + m_2)\, g &= \tau_1 \\ -m_2 l_2 c_2 \ddot{d}_1 + \left(m_2 l_2^2 + I_2\right) \ddot{\vartheta}_2 - m_2 g l_2 c_2 &= \tau_2 \end{aligned}$$

   The model can be written in the following form which is linear in the dynamic parameters:

$$\mathbf{Y}\left(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\right) \mathbf{\Pi} = \tau$$

   where the vector of dynamic parameters is expressed as:

$$\mathbf{\Pi} = \begin{bmatrix} m_1 + m_2 \\ m_2 l_2 \\ m_2 l_2^2 + I_2 \end{bmatrix}$$

   while the regressor matrix is:

$$\mathbf{Y} = \begin{bmatrix} \ddot{d}_1 + g & -c_2 \ddot{\vartheta}_2 & 0 \\ 0 & -c_2 \ddot{d}_1 - g c_2 & \ddot{\vartheta}_2 \end{bmatrix}$$

4. The linearity of the model in a set of dynamic parameters allows to setup experiments for the identification of such parameters. For a generic manipulator, explain what are the variables that need to be recorded during the experiments. With reference to the dynamic model of this exercise, is it possible to experimentally identify the mass of the first link?

   Since at each time instant the regressor matrix has to be computed, we need to record the joint positions, velocities and accelerations, along with the joint torques.
   For this specific manipulator it is not possible to identify the mass of the first link alone, we can only estimate the sum of the masses of the two links.

EXERCISE 2

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

   The kinematic calibration is a process, based on a series of measurements of the manipulator's end effector, that allows to obtain accurate estimates of the DH parameters. It is needed because of tolerances in mechanical building of components and in the assembly of the links and joints as well as for possible issues in encoder mounting

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta\mathbf{x} = \mathbf{\Phi}\Delta\zeta$$

   Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

   In this equation:
   $\Delta\mathbf{x} = \mathbf{x}_{act} - \mathbf{x}_{nom}$ are the deviations of the actual pose variables from the nominal ones (a 6×1 vector)
   $\mathbf{\Phi}$ is the calibration matrix, defined as:

$$\mathbf{\Phi} = \begin{bmatrix} \frac{\partial\mathbf{k}}{\partial\mathbf{a}} & \frac{\partial\mathbf{k}}{\partial\alpha} & \frac{\partial\mathbf{k}}{\partial\mathbf{d}} & \frac{\partial\mathbf{k}}{\partial\theta} \end{bmatrix}$$

   where $\mathbf{k}$ is the direct kinematic function, while $\mathbf{a}, \alpha, \mathbf{d}, \theta$ are the vectors of the DH parameters for the n joints. Matrix $\mathbf{\Phi}$ has 6 rows and $4n$ columns.
   $\Delta\zeta$ are the deviations of the DH parameters and are defined as:

$$\Delta\zeta = \begin{bmatrix} \Delta\mathbf{a} \\ \Delta\alpha \\ \Delta\mathbf{d} \\ \Delta\theta \end{bmatrix}$$

   $\Delta\zeta$ is a $4n \times 1$ vector.

3. Based on the above equation, explain how the kinematic calibration can be performed.

The equation:

$$\Delta \mathbf{x} = \mathbf{\Phi} \Delta \zeta$$

is a system of 6 equations in $4n$ unknowns. We need to perform a certain number l of experiments, each time changing the pose of the end-effector. Stacking the above equations referred to the various poses, we have:

$$\Delta \bar{\mathbf{x}} = \begin{bmatrix} \Delta \mathbf{x}_1 \\ \vdots \\ \Delta \mathbf{x}_l \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{\Phi}_1 \\ \vdots \\ \Delta \mathbf{\Phi}_l \end{bmatrix} \Delta \zeta = \bar{\mathbf{\Phi}} \Delta \zeta$$

this equation can be solved for $\Delta \zeta$ in a least squares form:

$$\Delta \zeta = \bar{\mathbf{\Phi}}^{\sharp} \Delta \bar{\mathbf{x}}$$

where $\bar{\mathbf{\Phi}}^{\sharp}$ is the left pseudoinverse of matrix $\bar{\mathbf{\Phi}}$.

The deviation $\Delta \zeta$ is then added to the nominal values of the DH parameters ζ. The process can be iterated until convergence under a certain threshold.

4. Consider now a kinematically redundant manipulator. Write the general solution of the inverse kinematics at velocity level. Is the pseudoinverse matrix that appears in this equation the same pseudoinverse of the kinematic calibration problem?

The forward kinematics at velocity level is written as:

$$\dot{\mathbf{r}} = \mathbf{J} \dot{\mathbf{q}}$$

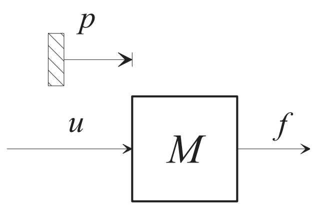
where $\dot{\mathbf{q}}$ are joint velocities, $\dot{\mathbf{r}}$ are task velocities and $\mathbf{J}$ is a Jacobian matrix. The solution of the inverse kinematics is written as:

$$\dot{\mathbf{q}} = \mathbf{J}^{\sharp} \dot{\mathbf{x}} + \left(\mathbf{I} - \mathbf{J}^{\sharp} \mathbf{J} \right) \dot{\mathbf{q}}_0$$

where $\mathbf{J}^{\sharp}$ is the right pseudoinverse of the Jacobian (in the kinematic calibration problem the left pseudoinverse is used).

EXERCISE 3

1. Consider a simple mass as in this picture:



Write the expression of an (explicit) impedance controller that can assign a prescribed and complete impedance relation.

To completely assign the impedance relation, we need to measure the interaction force. If this is the case, a suitable control law is:

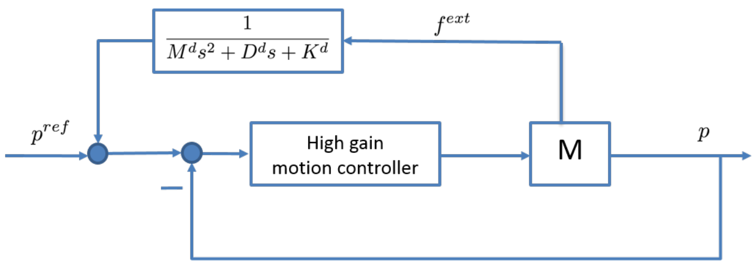
$$u = \frac{M}{M_d}(-k_1 p - k_2 v + f) - f$$

In this case, the following impedance relation is obtained:

$$M_d a + k_2 v + k_1 p = f$$

2. Still making reference to a single degree of freedom mechanism, sketch the block diagram of an admittance controller. What is the assumption that must be enforced on the motion control system in order to claim that the prescribed impedance is actually achieved?

The block diagram is sketched in the picture:



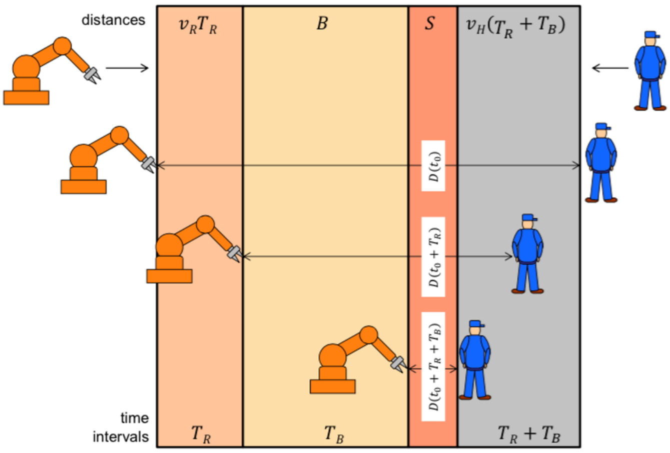
In order to have the prescribed impedance assigned, we need to assume a high bandwidth position controller, so that it guarantees that the output of the admittance controller is correctly tracked.

3. The admittance controller can be used to implement one of the possible collaborative modes between the robot and the human. Explain what is this mode and how admittance control can enable such collaborative mode. In particular, specify whether all the three elements of an impedance relation (mass, spring and damper) are used in this case.

The admittance control allows to implement the manual guidance. The force applied by the human when guiding the robot is acquired by the admittance controller and converted into suitable motion of the end-effector, so as to impose a generalized mass-spring-damper dynamical behaviour of the robot. Typically, the spring element is set to zero, to avoid the robot to come back to a rest position when the human leaves the end-effector.

4. The speed and separation monitoring is another collaborative mode. Making reference to the following picture, write the inequality that needs to be satisfied according to this safety standard. Is

the measurement of the human position needed in this standard and is it needed in the power and force limiting standard?



The inequality can be written as:

$$D(t_0) - v_R(T_r + T_B) - v_H(T_r + T_B) \geq S$$

meaning that the distance between human and robot has to be larger than the space that the robot and the human can cover, plus a minimum distance. The measurement of the human position is needed, in order to compute the distance with the robot, while in the power and force limiting standard it is not needed.