# Control of Industrial Robots 

Prof. Rocco

Jandary 22, 2024

## NAME:

UNIVERSITY ID NUMBER:
SIGNATURE:

## Warnings

- This file consists of $\mathbf{8}$ pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given either in English or in Italian.
- Solutions and answers must be given exclusively in the reserved space. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to hand this file only. Every other sheet you may hand will not be taken into consideration.


## EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:


Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.
2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms ${ }^{1}$ for this manipulator.
3. Write the complete dynamic model for this manipulator.

[^0]4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters.

## EXERCISE 2

1. The parametric form of a harmonic trajectory for kinematic scaling is given by:

$$
\sigma(\tau)=\frac{1}{2}(1-\cos (\pi \tau))
$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time $T$ and the total displacement $h$.
2. Consider the design of a harmonic trajectory from $q_{i}=10$ to $q_{f}=20$, with $\dot{q}_{\max }=20$ and $\ddot{q}_{\max }=20$. Find the minimum positioning time.
3. For the harmonic trajectory computed in this exercise, sketch the plot of the speed $\dot{q}(t)$, assuming that the trajectory starts at time $t_{i}=0$.
4. Still for the harmonic trajectory computed in this exercise, is the acceleration continuous in all time instants, including the initial and final times? What kind of issues discontinuities in the acceleration profile might imply in robotics?

## EXERCISE 3

1. Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:


Express the natural and the artificial constraints for this problem, and specify the selection matrix.
2. Sketch the block diagram of a hybrid force-position controller. What are possible sources of inconsistency in the adoption of such scheme?
3. Explain what an implicit force controller is and why it might be convenient with respect to an explicit solution.
4. Suppose now that along the translational $z$ direction an implicit force controller has to be designed. Sketch the block diagram of such controller and design it taking a bandwidth of $20 \mathrm{rad} / \mathrm{s}$.


[^0]:    ${ }^{1}$ The general expression of the Christoffel symbols is $c_{i j k}=\frac{1}{2}\left(\frac{\partial b_{i j}}{\partial q_{k}}+\frac{\partial b_{i k}}{\partial q_{j}}-\frac{\partial b_{j k}}{\partial q_{i}}\right)$

