Control of Industrial Robots

Prof. Rocco

September 7, 2023

NAME:
UNIVERSITY ID NUMBER:

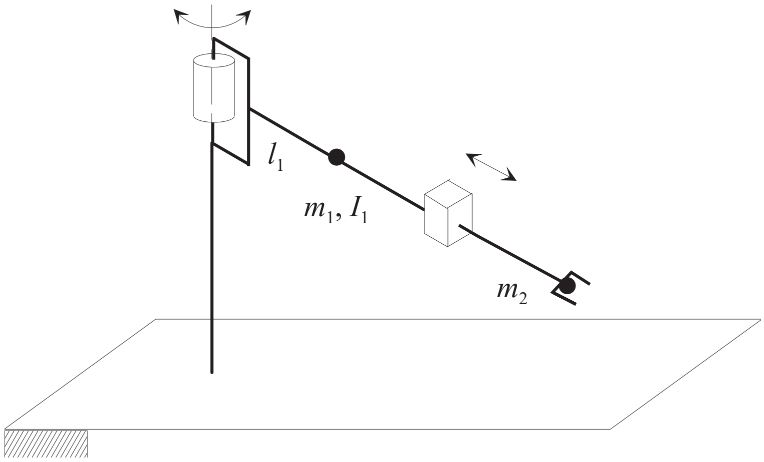
SIGNATURE: ______________________

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator, compute the expression of the derivative of the kinetic energy, exploiting the fact that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

EXERCISE 2

1. Explain what is the difference between the kinematic and the dynamic scaling of a trajectory

2. The parametric form of a cycloidal trajectory for kinematic scaling is given by:

$$\sigma(\tau) = \tau - \frac{1}{2\pi}\sin(2\pi\tau)$$

   Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time T and the total displacement h.

3. Consider the design of a cycloidal trajectory from $q_i = 0$ to $q_f = 10$, with $\dot{q}_{\max} = 30$ and $\ddot{q}_{\max} = 20$. Find the minimum positioning time.

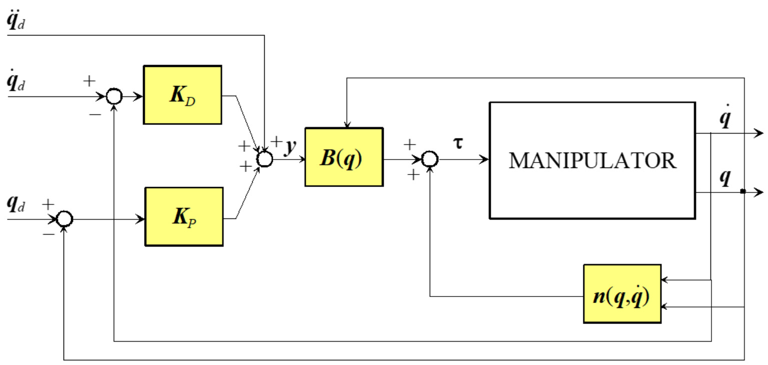
4. In the process of the dynamic scaling, the following relation is used, for each joint of the robot:

$$\tau_i(t) = \alpha_i\left(r(t)\right)\ddot{r}(t) + \beta_i\left(r(t)\right)\dot{r}^2(t) + \gamma_i\left(r(t)\right), \qquad i = 1, \ldots, n, \qquad t \in [0, T]$$

Explain the meaning of symbol r in this equation. Is it possible that some joint trajectories have a cycloidal dependence on time and some other joints a harmonic dependence on time and still obtain the above equation?

EXERCISE 3

1. Consider the control scheme sketched in the following picture:



Explain which control scheme it refers to and what is the result in terms of closed-loop dynamics that can be achieved with such control scheme.

2. In a two-link planar manipulator in the vertical plane with prismatic joints, the inertia matrix and the gravitational terms take the following expressions, respectively:

$$\mathbf{B} = \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} (m_1 + m_2)\, g \\ 0 \end{bmatrix}$$

Write the expression (equation by equation) of the control law for the control scheme of this exercise, specific for this manipulator.

3. Tune the two matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ in such a way that the dynamics of the error in the two joints is identical with two real eigenvalues at frequency 20 rad/s

4. Suppose now that there is uncertainty in the dynamic model of the robot. When is it appropriate to use a robust control system and when is it to use an adaptive one? Which one is a switching control law and why can this be a problem?