Control of Industrial Robots

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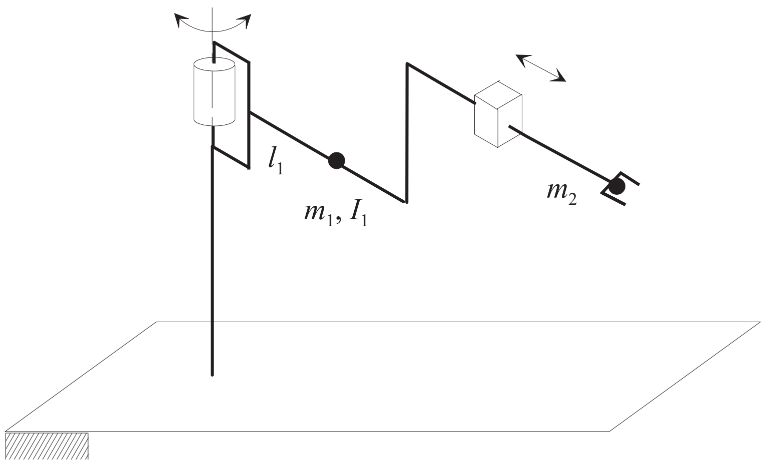
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the complete dynamic model for this manipulator.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters. Is it possible to identify the mass of the first link with experiments based on such model?

EXERCISE 2

1. Explain what is meant with kinematic scaling of a trajectory and write the general expression of a trajectory in the form (parameterized) which is used in such scaling.

2. The parametric form of a cycloidal trajectory for kinematic scaling is given by:

$$\sigma(\tau) = \tau - \frac{1}{2\pi}\sin(2\pi\tau)$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time T and the total displacement h.

3. Consider the design of a cycloidal trajectory from $q_i = 10$ to $q_f = 30$, with $\dot{q}_{\max} = 10$ and $\ddot{q}_{\max} = 20$. Find the minimum positioning time.

4. Suppose now that a harmonic trajectory is used for the same positioning. Do you expect a longer or a shorter minimum positioning time with respect to the previous one? Motivate your answer.

EXERCISE 3

Consider a kinematically redundant manipulator.

1. Write the general expression of the solutions of the inverse kinematics problem at velocity level, specifying what is the projection matrix and what is its role. Write also the cost function whose minimization leads to this general solution.

2. Express the solution in the form that includes a closed loop correction (kinematic control) and explain why this correction is used.

3. Discuss some criteria to select the joint velocities $\dot{\mathbf{q}}_0$ to be projected onto the null space of the Jacobian matrix.

4. Consider a robot with seven joints and a task that concerns only the position of the TCP. What is the size of the vector $\dot{\mathbf{q}}_0$ of joint velocities to be projected onto the null space of the Jacobian matrix and what is the size of the vector after such projection?