

# Control of Industrial Robots

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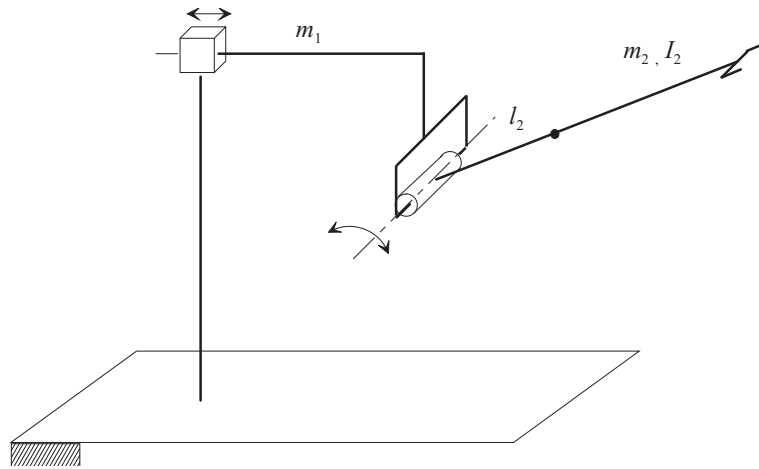
## Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.



## EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix  $\mathbf{B}(\mathbf{q})$  of the manipulator<sup>1</sup>

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<sup>1</sup>The cross product between vector  $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$  and  $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  is  $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  of the Coriolis and centrifugal terms<sup>2</sup> for this manipulator.

3. Check that matrix  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is skew symmetric.

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<sup>2</sup>The general expression of the Christoffel symbols is  $c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

4. For a generic manipulator, ignoring the gravitational terms and exploiting the skew symmetry of matrix  $\mathbf{N}$ , obtain an expression of the derivative with respect to time of the kinetic energy.

## **EXERCISE 2**

Consider a kinematically redundant manipulator.

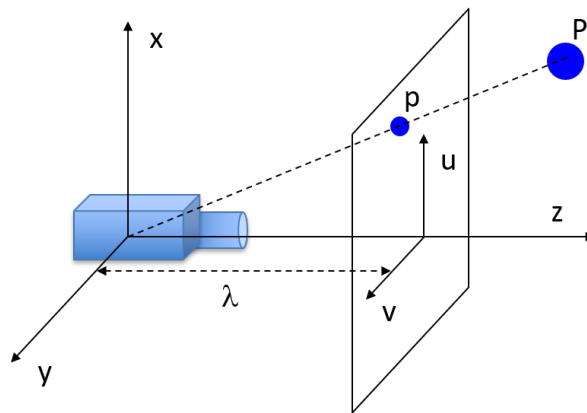
1. Write the general expression of the solutions of the inverse kinematics problem at velocity level.
2. Express the solution in the form that includes a closed loop correction (kinematic control) and explain why this correction is used.

3. Consider now the motion of the end effector along a linear path. Assigning to the natural coordinate  $s$  a cubic dependence on time, derive the expressions of the maximum speed and the maximum acceleration as functions of the displacement  $h$  and the positioning time  $T$ .
4. Assume that the length of the segment to cover is 1 m, the maximum linear velocity of the end effector is  $2m/s$  and the maximum linear acceleration  $4m/s^2$ . Compute the minimum positioning time, adopting a cubic dependence on time.

### EXERCISE 3

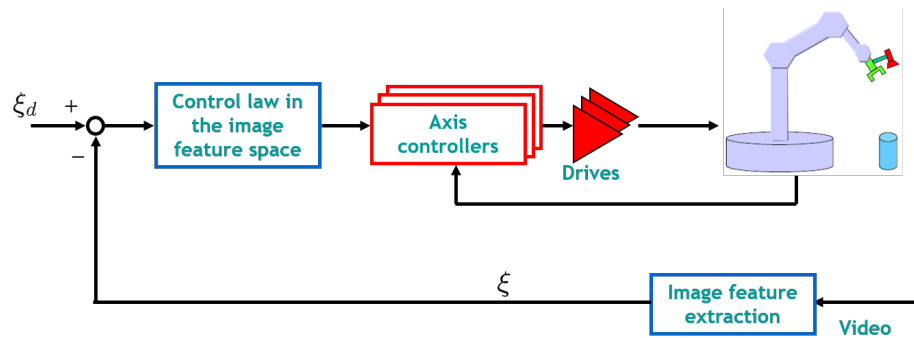
Consider a robot that uses a camera.

1. Explain what are the extrinsic and the intrinsic calibrations, making in particular reference to the notion of camera intrinsic matrix.
2. With reference to the following sketch, define what an image feature is and write the equations of the perspective projection method.



- Define the interaction matrix and the image Jacobian for a vision-based robotic system, in terms of the quantities that each of the two matrices relate.

- Consider now the following block diagram:



Is this a look-and-move or a visual servoing scheme? A position-based or an image-based scheme? Write an expression of the control law that can be used in this control scheme.