Control of Industrial Robots

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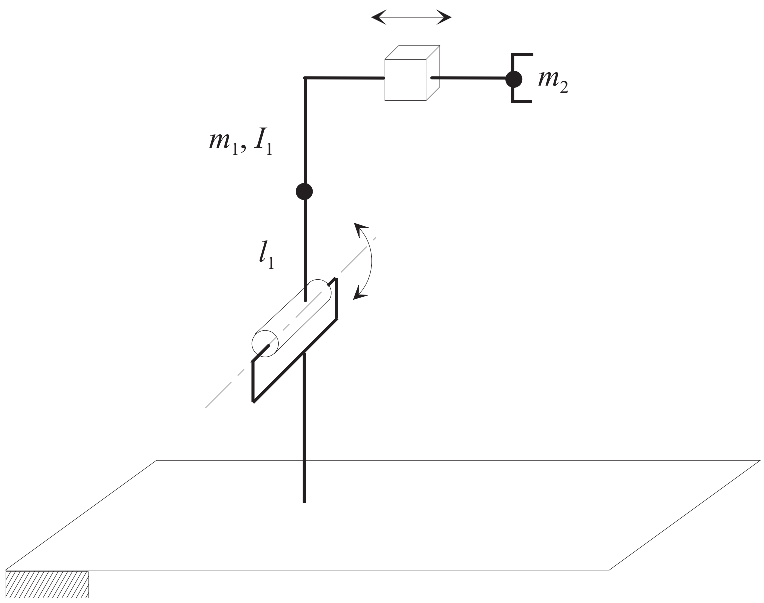
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



   Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Ignoring the gravitational terms, compute the dynamic model of this manipulator.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \dfrac{1}{2}\left(\dfrac{\partial b_{ij}}{\partial q_k} + \dfrac{\partial b_{ik}}{\partial q_j} - \dfrac{\partial b_{jk}}{\partial q_i}\right)$

4. Write the dynamic model in a linear form with respect to a set of dynamic parameters.

EXERCISE 2

1. Explain what is the difference between the kinematic and the dynamic scaling of a trajectory

2. The parametric form of a cycloidal trajectory for kinematic scaling is given by:

$$\sigma(\tau) = \tau - \frac{1}{2\pi}\sin(2\pi\tau)$$

   Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time T and the total displacement h.

3. Consider the design of a cycloidal trajectory from $q_i = 0$ to $q_f = 10$, with $\dot{q}_{\max} = 20$ and $\ddot{q}_{\max} = 30$. Find the minimum positioning time.

4. In the process of the dynamic scaling, the following relation is used, for each joint of the robot:

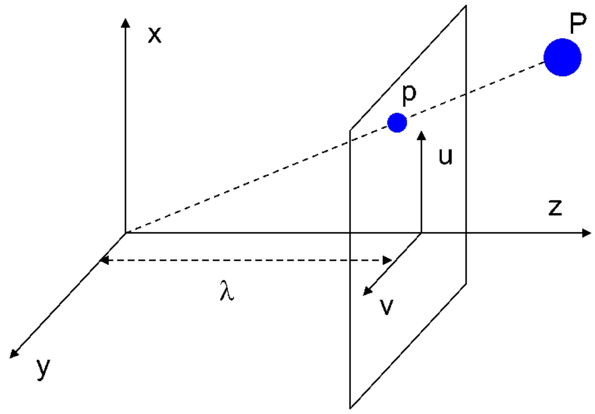
$$\tau_i(t) = \alpha_i\left(\sigma(t)\right)\ddot{\sigma}(t) + \beta_i\left(\sigma(t)\right)\dot{\sigma}^2(t) + \gamma_i\left(\sigma(t)\right), \qquad i = 1, \ldots, n, \qquad t \in [0, T]$$

Explain the meaning of symbol σ in this equation. Out of the three terms in the right hand side, which ones scale with time?

EXERCISE 3

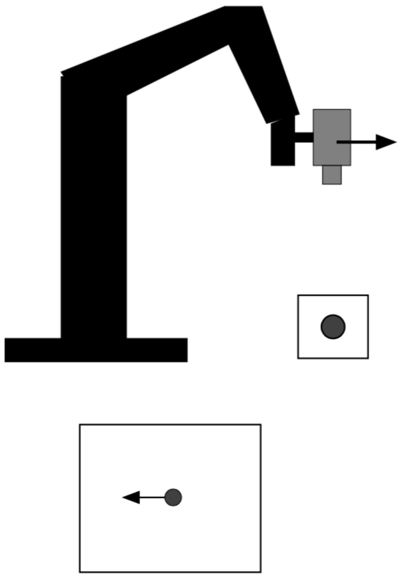
Consider the control of a manipulator with vision sensors.

1. Explain what is the “perspective projection” method and, making reference to the following picture, write the related formulas.



2. Making reference to the following picture, where a single image point is considered, explain what is the interaction matrix in the context of visual control, specifying precisely:

   - the variables that are related by the interaction matrix
   - the size of the interaction matrix
   - the variables upon which the interaction matrix depends



3. Suppose now that there are n image points: explain how to build the interaction matrix and what is the size of such matirx in this case.

4. Explain what is the image Jacobian and what is its relation with the interaction matrix.