# Control of Industrial Robots

Prof. Rocco

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NAME:
UNIVERSITY ID NUMBER:

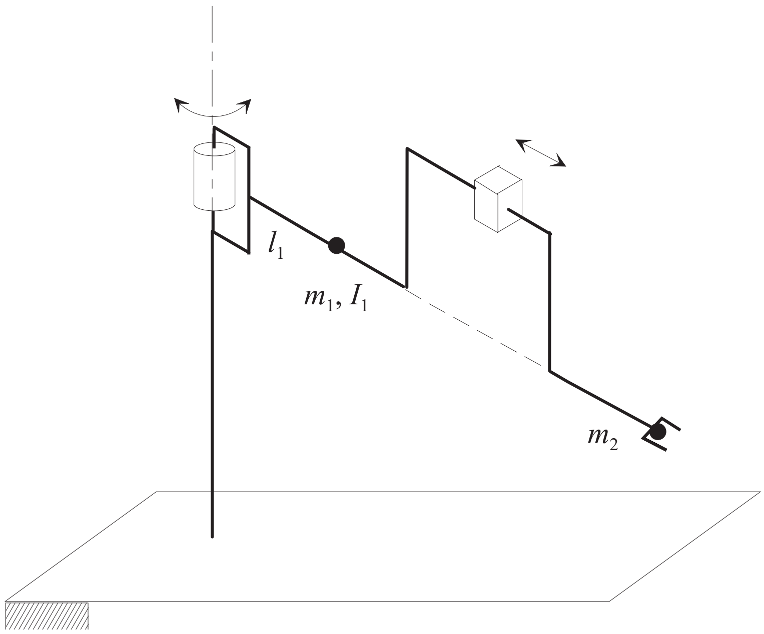
SIGNATURE: ______________________________

**Warnings**

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

**EXERCISE 1**

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:




Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the dynamic model for this manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator without gravitational and friction effects, show that the equation:

$$\dot{\mathbf{q}}^T \left( \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \right) \dot{\mathbf{q}} = 0$$

is valid for any choice of the Coriolis and centrifugal matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

**EXERCISE 2**

Consider a kinematically redundant manipulator.

1. Write the general expression of the solutions of the inverse kinematics problem at velocity level.

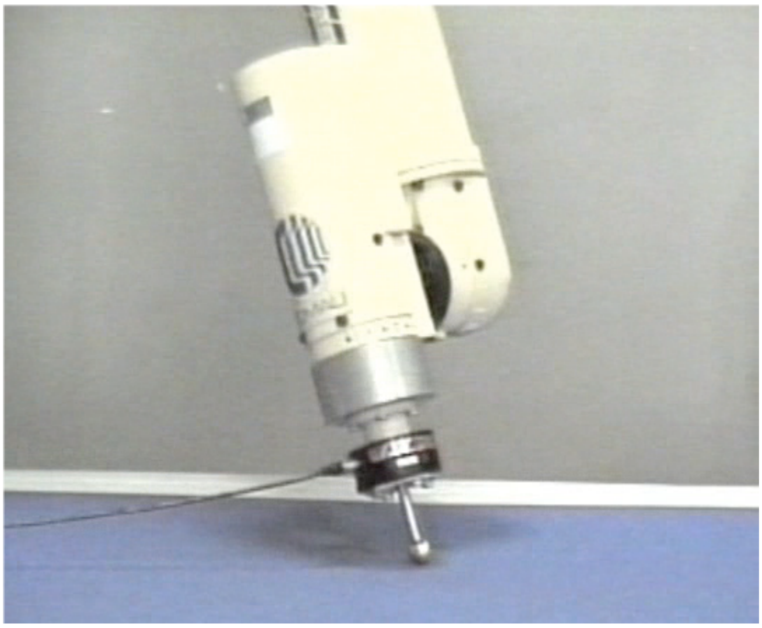
2. Consider the weighted pseudo inverse method for the solution of the inverse kinematics of a redundant manipulator. Write the expression of the cost function and discuss a criterion to select the weights.

3. The inverse kinematics at velocity level for a redundant manipulator is often implemented in closed-loop. Explain the reason for this and sketch the block diagram for such a closed-loop scheme.

4. Consider now motion planning of the end-effector position. If the end-effector task is expressed in terms of position only, what is the minimum number of joints for the manipulator to be redundant with respect to this task?

**EXERCISE 3**

Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:



1. Assume a point contact and draw a contact frame directly on the picture. Based on this frame and neglecting angular velocities and moments, express the natural and the artificial constraints for this problem.

2. Write the expression of the selection matrix for this problem, explaining the meaning of such matrix.

3. Suppose now that along the force controlled direction an explicit force controller has to be designed. Determine the expression of such controller, taking a bandwidth of 25 rad/s.

4. If you want to move the robot along the contact plane by hand, applying suitable forces, what kind of controller would you adopt? What is the difference between this control problem and the control problem of the contact force, addressed in the previous step?