Control of Industrial Robots

PROF. ROCCO

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NAME:
UNIVERSITY ID NUMBER:

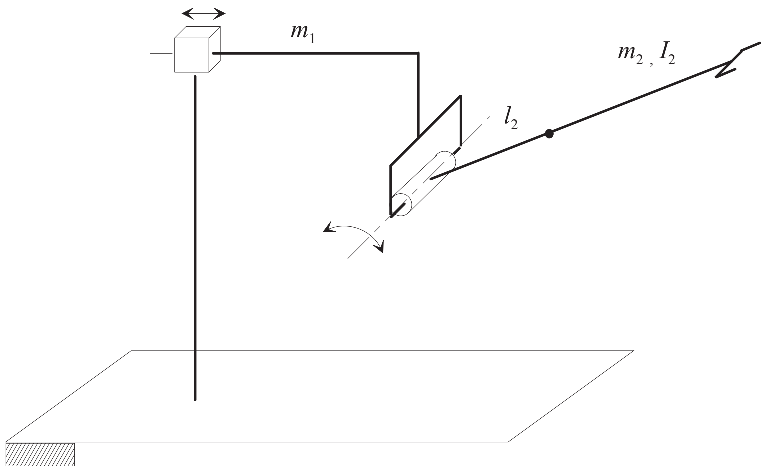
SIGNATURE: ____________________

Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1]

[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

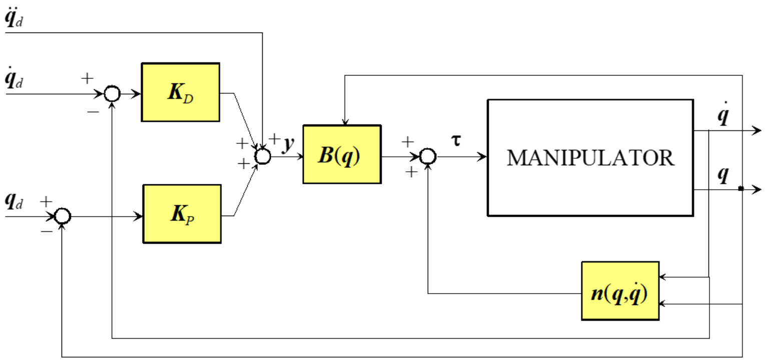
3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Mention one situation where the skew symmetry of matrix $\mathbf{N}$ is used.

EXERCISE 2

1. Consider the block diagram sketched in the following picture:



Explain which control scheme it refers to and what is the result in terms of closed-loop dynamics that can be achieved with such control scheme.

2. Consider now a generic two-link manipulator, for which the inertia matrix, the matrix of the Coriolis and centrifugal terms, and the vector of the gravitational terms can be written as follows, respectively:

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \mathbf{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Write the expression (equation by equation) of the control law for the control scheme of this exercise, for this two d.o.f. manipulator.

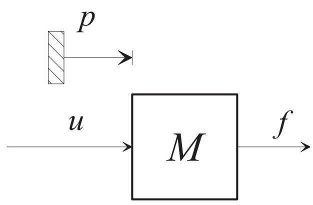
3. Tune the two matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ in such a way that the dynamics of the error in the two joints is identical, with two real eigenvalues at frequencies 10 rad/s and 30 rad/s.

4. Suppose now that you have only a partial knowledge of the dynamic model of the robot. Does the closed loop dynamics change with respect to the previous discussion? Is it still possible to guarantee stability in the closed loop system?

EXERCISE 3

1. Explain what is an "impedance" in general and how the concept of impedance can be used in robotics.

2. Consider a simple mass as in this picture:



Write the expression of an (explicit) impedance controller that can assign a prescribed and complete impedance relation.

3. Still making reference to a single degree of freedom mechanism, sketch the block diagram of an admittance controller. What is the assumption that must be enforced on the motion control system in order to claim that the prescribed impedance is actually achieved?

4. Suppose that a force sensor is not available. Still making reference to a single degree of freedom mechanism, explain how the external force can be estimated.