Control of Industrial Robots

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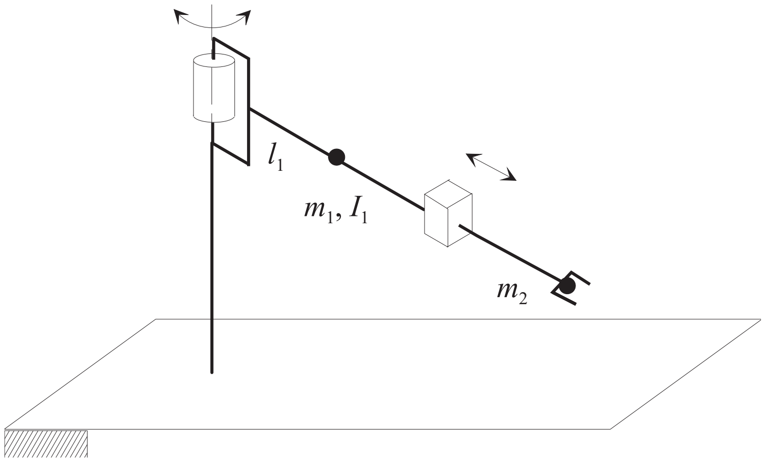
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator. Is this the only possible expression that matrix $\mathbf{C}$ can take?

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Consider now the expression $\dot{\mathbf{q}}^T \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} = 0$. Explain whether this result can be obtained only with a matrix $\mathbf{C}$ computed from the Christoffel symbols or if it is general.

EXERCISE 2

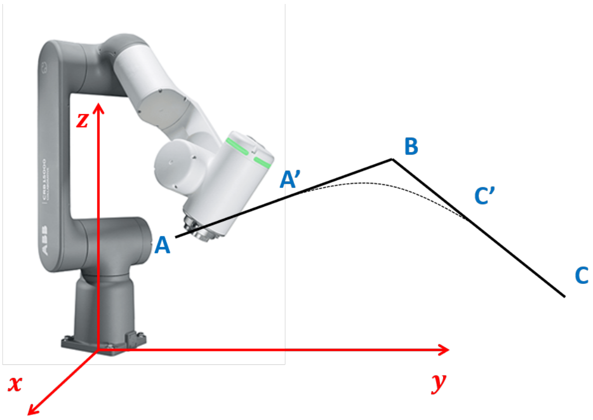
1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$\begin{array}{ccccc} t_1 = 0 & t_2 = 3 & t_3 = 5 & t_4 = 7 & t_5 = 10 \\ q_1 = 10 & q_2 = 0 & q_3 = 30 & q_4 = 60 & q_5 = 75 \end{array}$$

Assume that you want to use cubic polynomials in each interval. Assign suitable values to the speed at the intermediate points.

2. Suppose now that you want to use the method of the splines to interpolate the previous points. Will you compute the speeds in the intermediate points in the same way as before, or will you use a different method? In case, specify which method, without going into any mathematics.

3. The concatenation of linear paths is a common practice in robotic motion planning. Making reference to the following picture, and without going through all the mathematics, explain what are the considerations that can be done in order to design such concatenation.



4. Consider the following instructions in the ABB robot programming language:

```
MoveL p10, v200, fine;
MoveL p20, v200, z20;
MoveL p30, v200, fine;
MoveL p40, v200, z50;
MoveL p10, v200, fine;
```

Briefly explain what is the meaning of such instructions.

EXERCISE 3

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta \mathbf{x} = \boldsymbol{\Phi} \Delta \zeta$$

   Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

3. In the kinematic calibration, the pseudoinverse of a matrix is used. Explain precisely what matrix is this and whether the right or left pseudoinverse is used.

4. In order to find the solution of the inverse kinematics for a redundant robot, a pseudoinverse matrix is used as well. Explain what matrix is this and whether the right or left pseudoinverse is used in this case.