# Control of Industrial Robots 

Prof. Rocco

June 18, 2024

## NAME:

UNIVERSITY ID NUMBER:
SIGNATURE:

## Warnings

- This file consists of $\mathbf{8}$ pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given either in English or in Italian.
- Solutions and answers must be given exclusively in the reserved space. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to hand this file only. Every other sheet you may hand will not be taken into consideration.


## EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:


Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator ${ }^{1}$.
${ }^{1}$ The cross product between vector $a=\left[\begin{array}{c}a_{1} \\ a_{2} \\ a_{3}\end{array}\right]$ and $b=\left[\begin{array}{l}b_{1} \\ b_{2} \\ b_{3}\end{array}\right]$ is $c=a \times b=\left[\begin{array}{c}a_{2} b_{3}-a_{3} b_{2} \\ a_{3} b_{1}-a_{1} b_{3} \\ a_{1} b_{2}-a_{2} b_{1}\end{array}\right]$
2. Compute the vector of the gravitational terms for this manipulator.
3. Write the expression of the PD + gravity compensation control law for this manipulator.
4. What is the theoretical property guaranteed by a PD + gravity compensation control scheme?

## EXERCISE 2

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$
\begin{array}{ccccc}
t_{1}=0 & t_{2}=3 & t_{3}=5 & t_{4}=7 & t_{5}=10 \\
q_{1}=30 & q_{2}=0 & q_{3}=20 & q_{4}=50 & q_{5}=65
\end{array}
$$

Assume that you want to use cubic polynomials in each interval. Assign suitable values to the speed at the intermediate points.
2. Consider now the interpolation of such points by a single polynomial of suitable degree. What are the issues that suggest not to use this solution?
3. The concatenation of linear paths is a common practice in robotic motion planning. Making reference to the following picture, and without going through all the mathematics, explain what are the considerations that can be done in order to design such concatenation.

4. Consider the following instructions in the ABB robot programming language:

```
MoveL p10, v200, fine;
MoveL p20, v200, z20;
MoveL p30, v200, fine;
MoveL p40, v200, z50;
MoveL p10, v200, fine;
```

Briefly explain what is the meaning of such instructions.

## EXERCISE 3

Consider a kinematically redundant manipulator.

1. Explain when a robot is kinematically redundant and what are possible advantages of kinematic redundancy.
2. A common solution of the inverse kinematics problem for a redundant robot is to use the pseudoinverse of the Jacobian. Explain where the solution with the pseudoinverse comes form (what is the optimization problem solved and how it is solved).
3. Explain what is a weighted pseudo inverse, what is the optimization problem a weighted pseudo inverse solves and why it might be used to solve the inverse kinematics for a redundant robot.
4. Explain what are the null space motions for a redundant robot and what is the expression of the projection matrix.
