Control of Industrial and Mobile Robots

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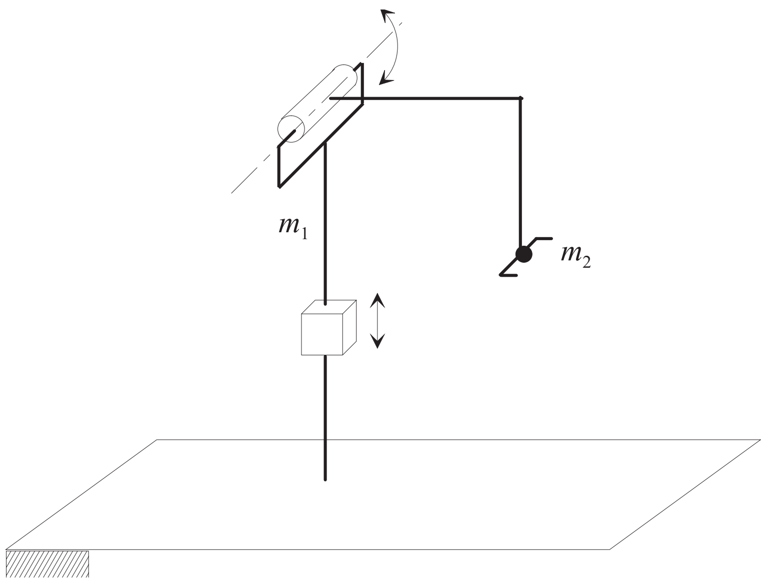
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the vector of the gravitational terms for this manipulator.

3. Write the expression of the PD + gravity compensation control law for this manipulator.

4. What is the theoretical property guaranteed by a PD + gravity compensation control scheme?

EXERCISE 2

1. Explain why for the kinematic scaling of trajectories it is possible to consider each joint separately, while for the dynamic scaling this is not possible.

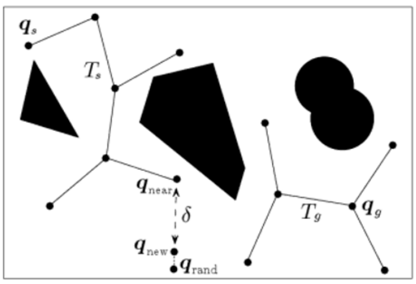
2. Consider the following equation:

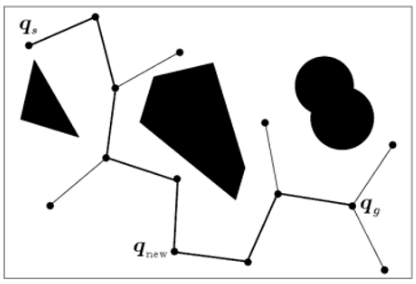
$$\tau_i(t) = \alpha_i\left(r(t)\right)\ddot{r}(t) + \beta_i\left(r(t)\right)\dot{r}^2(t) + \gamma_i\left(r(t)\right)$$

   Explain whether such equation is used in the kinematic scaling or in the dynamic scaling and define all symbols used in the equation.

3. Assume now that, in a robot that is not affected by gravity, trajectories have been planned such that the torque of one joint exceeds its limit by 44% (the torques of the other joints are within their limits). Explain how the trajectory can be scaled and what should be the scaling factor.

4. With reference to the following picture, discuss the RRT method for the path planning with obstacle avoidance for robotic manipulators.





EXERCISE 3

1. Are the following sentences true or false?

		T	F
(A)	A bilateral kinematic constraint is characterized by an equality constraint	☐	☐
(B)	The existence of k holonomic constraints implies the existence of k kinematic constraints	☐	☐
(C)	In the derivation of the kinematic model the choice of vectors $\mathbf{g}_1(\mathbf{q}), \ldots, \mathbf{g}_m(\mathbf{q})$ is unique	☐	☐
(D)	Holonomic constraints cause a loss of accessibility	☐	☐

2. Assuming that $A^T(\mathbf{q})\,\dot{\mathbf{q}} = 0$ is the Pfaffian form of the following system of kinematic constraints

$$3\dot{q}_1 + \cos(q_1)\,\dot{q}_2 - \dot{q}_3 = 0 \qquad 7\sin(q_1)\,\dot{q}_1 - \dot{q}_2 + \dot{q}_3 = 0$$

where $\mathbf{q} = [q_1\ q_2\ q_3\ q_4]^T$, and

$$g_1(\mathbf{q}) = \begin{bmatrix} 1-\cos(q_1) & 3+7\sin(q_1) & 3+7\cos(q_1)\sin(q_1) & 0 \end{bmatrix}^T \qquad g_2(\mathbf{q}) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$$

are two vectors in the null space of $A^T(\mathbf{q})$, demonstrate that the system of two constraints is holonomic.

3. Does the following kinematic model

$$\dot{\mathbf{q}} = g_1(\mathbf{q})\, u_1 + g_2(\mathbf{q})\, u_2$$

where $g_1(\mathbf{q})$ and $g_2(\mathbf{q})$ are the vectors introduced in the previous question, describe the motion of the mobile robot characterized by the system of two constraints?

4. The following two codes (A and B) represent two different ways of coding the periodic loop of a ROS node implementing a kinematic simulator. Which of the two is correct? Clearly motivate the answer.

Listing 1: A

```
void simulator::RunPeriodically(void)
{
    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        usleep(1000);
    }
}
```

Listing 2: B

```
void simulator:RunPeriodically(float
    Period)
{
    ros::Rate LoopRate(1.0/Period);

    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        LoopRate.sleep();
    }
}
```

EXERCISE 4

1. Are the following sentences true or false?

		T	F
(A)	The goal region is a subspace of the configuration space, i.e., $\mathcal{Q}_{goal} \subset \mathcal{Q}$	☐	☐
(B)	A feasible path is a continuous and collision-free path	☐	☐
(C)	PRM/PRM* are multi-query algorithms, RRT/RRT* are single-query algorithms	☐	☐
(D)	In optimal sampling-based planning the cost function must be monotonic and unbounded	☐	☐

2. Write the pseudocode of the function used by RRT* planner to locally connect along a minimum path.

3. Write the pseudocode of the function used by RRT* planner to rewire the tree.

4. Give the formal definition of the cost function used by RRT* planner, and explain which assumption is usually introduced on the cost function in order to simplify the planning algorithm.