Control of Industrial and Mobile Robots

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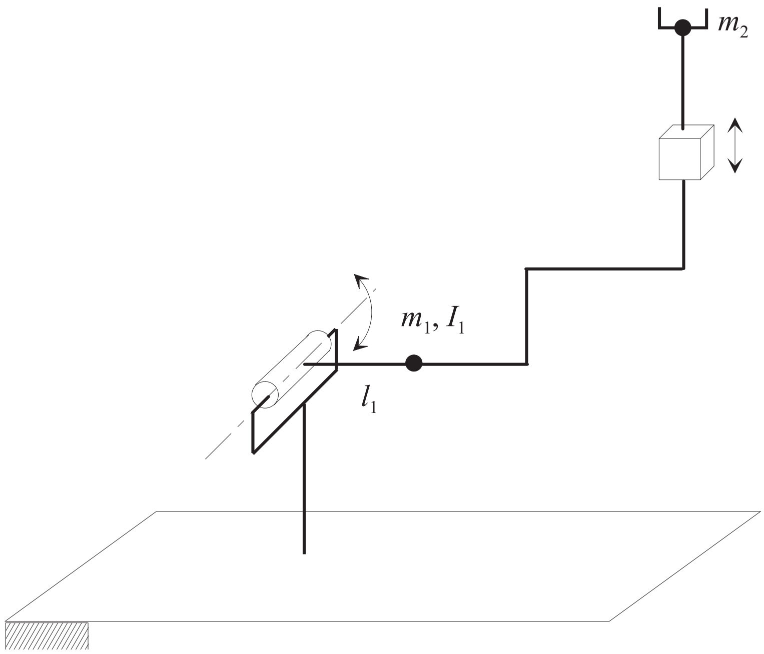
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

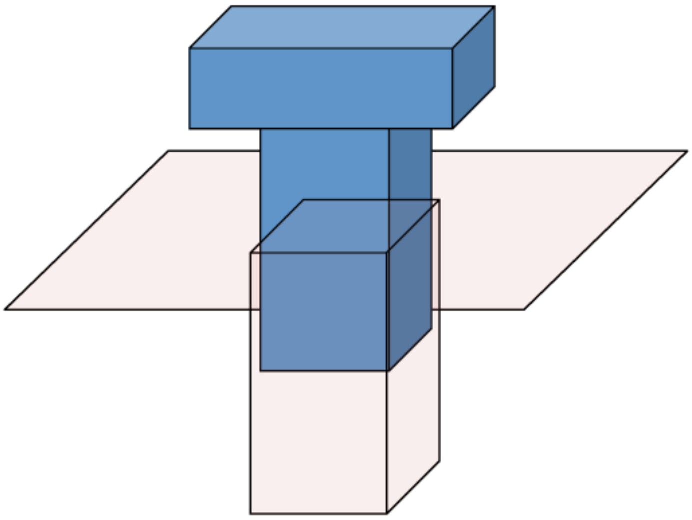
3. Ignoring the gravitational terms, write the complete dynamic model for this manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Explain what are the direct and the inverse dynamics. With reference to the model obtained in the previous step, define what are the inputs and the outputs of the direct and of the inverse dynamics algorithms.

EXERCISE 2

Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:



1. Select a convenient contact frame and identify the natural and the artificial constraints.

2. Write the expression of the selection matrix for this problem, explaining the meaning of such matrix.

3. Suppose now that along one of the force controlled directions an explicit force controller has to be designed. Determine the expression of such controller, taking a bandwidth of 30 rad/s.

4. Define what an implicit controller of the force is and explain why it might be more convenient to use than an explicit controller. Do you need to know precisely the stiffness constant of the environment to tune such controller?

EXERCISE 3

1. Are the following sentences true or false?

		T	F
(A)	In a system of kinematic constraints, if one is nonholonomic the system is nonholonomic	☐	☐
(B)	The Lie Bracket of two vector fields is a vector field perpendicular to the first one	☐	☐
(C)	If the kinematic model is not controllable, the corresponding kinematic constraints are nonholonomic	☐	☐
(D)	In a front-wheel drive bicycle the heading dynamics are described by $\dot{\theta} = v/\ell \cdot \sin\phi$	☐	☐

2. A kinematic model $\dot{\mathbf{q}} = \sum_{j=1}^{m} \mathbf{g}_j(\mathbf{q})\, u_j$, where $m = n - k$, $\mathbf{q} \in \mathbb{R}^n$, is generated by k kinematic constraints. Assuming that the model is not controllable, is the set of constraints holonomic or nonholonomic?

3. Consider the kinematic model of a unicycle robot, whose inputs are v and ω, and whose states are x, y, and θ. Due to the presence of internal controllers the relation between the velocity commands v_{cmd}, ω_{cmd} and the velocities v, ω is given be

$$v(s) = \frac{1}{1 + sT_v} v_{cmd}(s) \qquad \omega(s) = \frac{1}{1 + sT_\omega} \omega_{cmd}(s)$$

Complete the following C++ function used by odeint to simulate the model, assuming that `u1`, and `u2` are two variables of the class `simulator` representing v_{cmd}, and ω_{cmd}, respectively.

```
void simulator::simulator_ode(const state_type &state, state_type &
    dstate, double t)
{



}
```

4. The following two codes (A and B) represent two different ways of implementing the node periodic loop of the simulator. Which of the two is correct? Clearly motivate the answer.

Listing 1: A

```
void simulator::RunPeriodically(void)
{
    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        usleep(1000);
    }
}
```

Listing 2: B

```
void simulator:RunPeriodically(float
    Period)
{
    ros::Rate LoopRate(1.0/Period);

    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        LoopRate.sleep();
    }
}
```

EXERCISE 4

1. Are the following sentences true or false?

		T	F
(A)	To transform the canonical model into two independent integrators, point P can be selected in any position except for the wheel axis	☐	☐
(B)	To control an omnidirectional robot any linear controller can be used	☐	☐
(C)	Depending on the environment, an STLA robot may not be able to reach all configurations of the free space	☐	☐
(D)	Discretizing the unicycle model with Euler method, the relations giving the position are exact, the one giving the orientation is approximated	☐	☐

2. Consider the design of the trajectory tracking controller for a mobile robot that has been already provided with a linearizing controller. Write the model on which the trajectory tracking controller is designed, and the expression of a trajectory tracking controller based on a proportional control plus velocity feedforward.

3. With reference to the controller in the previous question, according to which criterion(s) is the bandwidth of this controller chosen and, therefore, the value of its parameters?

4. Consider the odometric localization of a differential drive robot. Assuming that $\Delta\phi_{R_k}$ and $\Delta\phi_{L_k}$ are the encoder measurements during the k-th sampling interval of duration T_s, R the wheel radius, and d the distance between the wheel contact points, write the relations that allow to estimate the pose of the robot using the second-order Runge-Kutta integration method.