Control of Industrial and Mobile Robots

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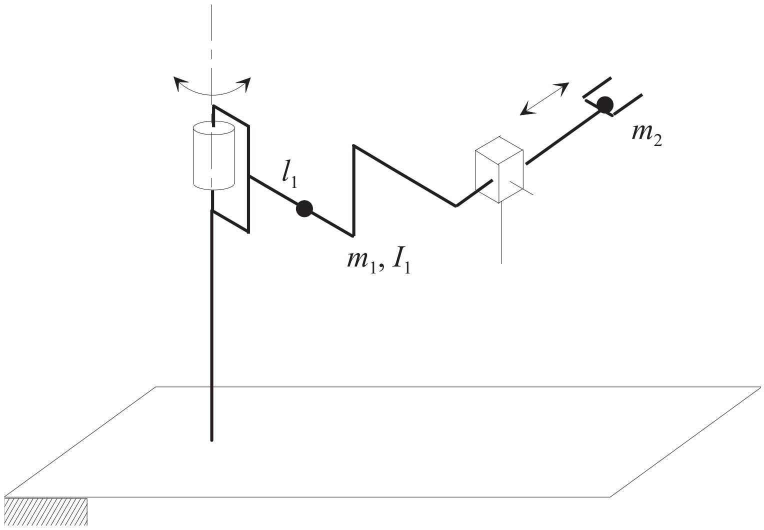
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the complete dynamic model for this manipulator and specify whether this model depends on both joint positions, both joint velocities, and both joint accelerations.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters. Is it possible through suitable experiments to identify the value of the mass of the first link m_1?

EXERCISE 2

1. Write the parametric expression (in terms of a natural coordinate s) of a segment in space, used for planning a linear path.

2. Show that, in the general case, the absolute value of the time derivative of the natural coordinate s is the norm of the linear velocity of the end-effector.

3. Consider now the planning along a linear path. Assume that the length of the segment to cover is 0.5 m and that the maximum linear velocity of the end effector is 1.5 m/s. Compute the minimum positioning time, if a cycloidal dependence on time [2] of the natural coordinate is used.

4. Explain what is an artificial potential method in the context of path planning with obstacle avoidance. What is a possible issue with this method?

[2]The normalized expression of a cycloidal trajectory is $\sigma(\tau) = \tau - \frac{1}{2\pi}\sin(2\pi\tau)$

EXERCISE 3

Consider the following system of kinematic constraints

$$\dot{q}_1 + \cos(q_1)\,\dot{q}_2 + \dot{q}_3 = 0$$
$$\sin(q_1)\,\dot{q}_1 + \dot{q}_2 - \dot{q}_3 = 0$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$.

1. Using the necessary and sufficient condition, determine if the first constraint, considered as an independent constraint, is holonomic or nonholonomic.

2. Using the necessary and sufficient condition, determine if the second constraint, considered as an independent constraint, is holonomic or nonholonomic.

3. Assuming that $A^T(\mathbf{q})\,\dot{\mathbf{q}} = 0$ is the Pfaffian form of the system of two constraints, and

$$g_1(\mathbf{q}) = \begin{bmatrix} 1+\cos(q_1) \\ -(1+\sin(q_1)) \\ \cos(q_1)\sin(q_1)-1 \\ 0 \end{bmatrix} \qquad g_2(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

are two vectors in the null space of $A^T(\mathbf{q})$, demonstrate that the system of two constraints is holonomic.

4. Does the following kinematic model

$$\dot{\mathbf{q}} = g_1(\mathbf{q})\, u_1 + g_2(\mathbf{q})\, u_2$$

where $g_1(\mathbf{q})$ and $g_2(\mathbf{q})$ are the vectors introduced in the previous step, describe the motion of the mobile robot characterized by the system of two constraints?

EXERCISE 4

Consider the design of the trajectory tracking controller of a robot described by the rear-wheel-drive bicycle model.

1. Write the relations that allow to transform the bicycle model into the canonical simplified model. Under which assumptions do these relations hold?

2. Considering a point P, located along the linear velocity vector v at a distance ε from the wheel contact point, write the analytical expression of a feedback linearizing controller for the canonical simplified model.

3. Consider the implementation of a controller as a ROS node. The following two codes (A and B) represent two different ways of implementing the node periodic loop. Which of the two is the correct implementation for a controller node? Clearly motivate the answer.

Listing 1: A

```
void controller::RunPeriodically(void)
{
    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        usleep(1000);
    }
}
```

Listing 2: B

```
void controller:RunPeriodically(float
    Period)
{
    ros::Rate LoopRate(1.0/Period);

    while (ros::ok())
    {
        PeriodicTask();
        ros::spinOnce();
        LoopRate.sleep();
    }
}
```

4. Explain the difference between implementing a trajectory tracking controller where the function that computes the control variable is called in `PeriodicTask`, and one where it is called by the callback that receives the robot's position measurements. Why is it wrong to call it in the callback?