

Control of Industrial and Mobile Robots

PROF. ROCCO, BASCETTA

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NAME:

UNIVERSITY ID NUMBER:

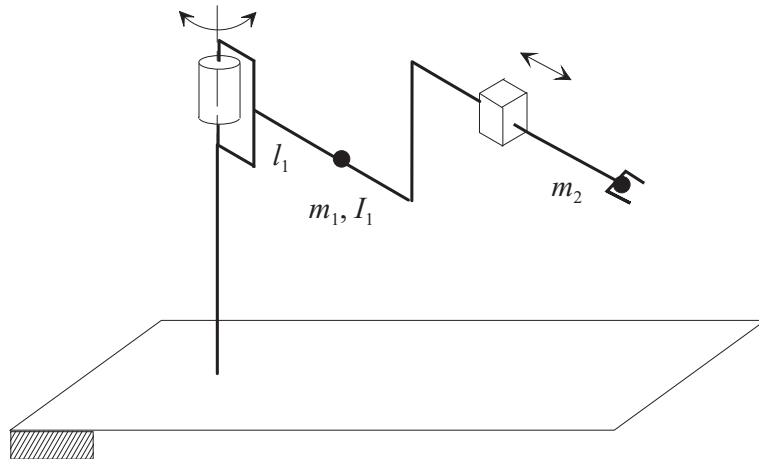
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Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms¹ for this manipulator.

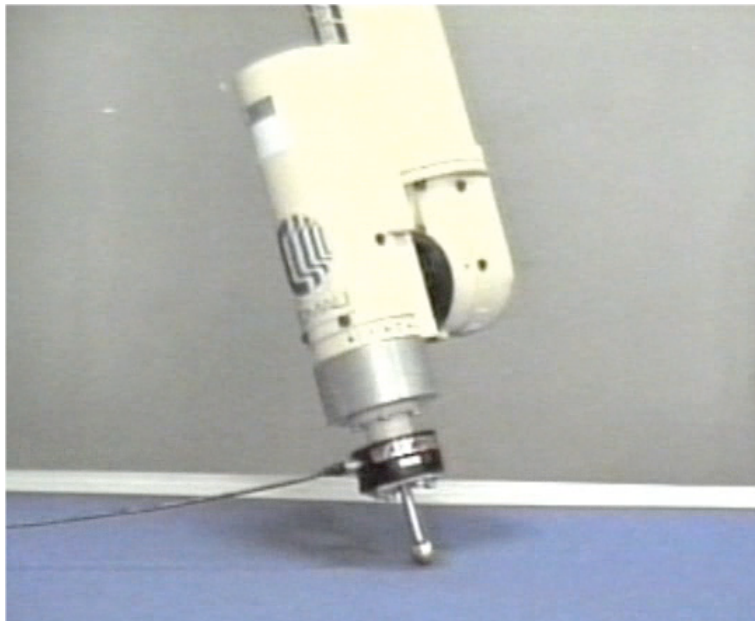
3. Write the complete dynamic model for this manipulator.

¹The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2} \left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

4. Write the equations of an inverse dynamics controller for this robot. What are the dynamic parameters that need to be identified in order to implement such controller?

EXERCISE 2

Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:



1. Assume a point contact and draw a contact frame directly on the picture. Based on this frame and neglecting angular velocities and moments, express the natural and the artificial constraints for this problem.

2. Write the expression of the selection matrix for this problem, explaining the meaning of such matrix.

3. Sketch the block diagram of a hybrid force/position controller. What happens if there is friction at the contact between the tool and the surface?

4. Suppose now that along the force controlled direction an explicit force controller has to be designed. Determine the expression of such controller, taking a bandwidth of 30 rad/s.

EXERCISE 3

1. Given the kinematic constraint

$$\dot{q}_1 + 5\dot{q}_4 = 0$$

where $\mathbf{q} \in \mathbb{R}^4$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Given the kinematic constraint

$$q_1\dot{q}_2 + 5\dot{q}_3 = 0$$

where $\mathbf{q} \in \mathbb{R}^4$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Is the system of constraints

$$\begin{aligned}\dot{q}_1 + 5\dot{q}_4 &= 0 \\ q_1\dot{q}_2 + 5\dot{q}_3 &= 0\end{aligned}$$

holonomic or nonholonomic? Motivate the answer analysing the accessibility distribution. Note that $\text{rank}(A^T(\mathbf{q})) = 2$, and two vectors in the null space of $A^T(\mathbf{q})$ are

$$g_1(\mathbf{q}) = \begin{bmatrix} 0 \\ 5 \\ -q_1 \\ 0 \end{bmatrix} \quad g_2(\mathbf{q}) = \begin{bmatrix} 5 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

4. Assume that `\state` is the topic used by a simulator to publish the state of a bicycle robot, and `state_publisher` is a variable of the node class, of type `ros::Publisher`, representing a publisher for that topic with type `std_msgs::Float64MultiArray`. Write the lines of code used by the node to publish the state $x = 5$, $y = 2$, $\theta = 0.2$, and $\phi = 0.1$.

EXERCISE 4

1. Are the following sentences true or false?

	T	F
(A) In a planning problem the initial condition is a configuration that must belong to $\mathcal{Q}_{free}$	☐	☐
(B) PRM and sPRM can be implemented considering different choices for the set of “near” nodes (Near, kNearest, etc.)	☐	☐
(C) In optimal sampling-based planners the cost function must be monotonic and unbounded	☐	☐
(D) Kinodynamic constraints can be classified into: kinematic constraints, dynamic constraints, and actuator constraints	☐	☐

2. Write the pseudocode of RRG planner.

3. Considering an optimal planner, how can you choose the number of nodes N , adopting the best compromise between solution optimality and computational complexity?
Clearly justify your answer.

4. Consider an optimal kinodynamic planner, like for example kinodynamic RRT*, write the mathematical formulation of the steering problem for a unicycle robot, considering hard constraints on the linear acceleration and the angular velocity (that should stay in a range between a given minimum and maximum value), a cost function representing minimum time with a weight on actuation effort, and an initial and final state that allow to specify robot pose and velocity.