

Control of Industrial and Mobile Robots

PROF. ROCCO, BASCETTA

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NAME:

UNIVERSITY ID NUMBER:

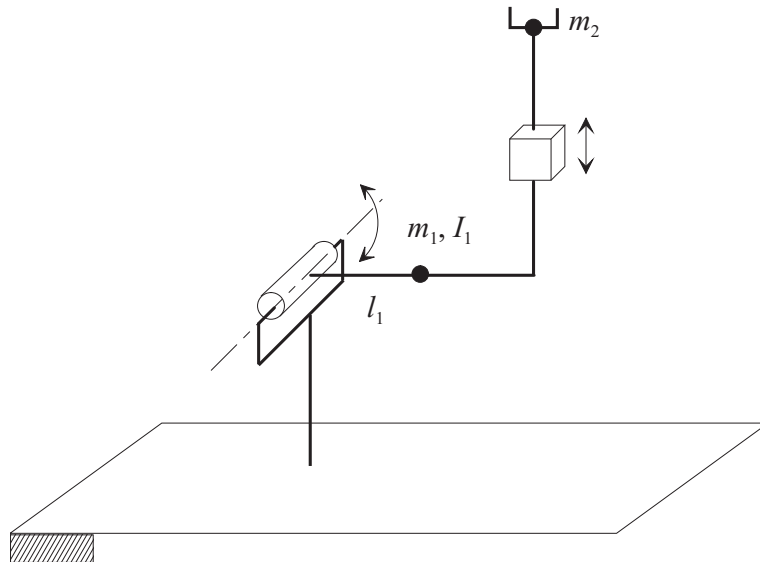
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Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms¹ for this manipulator.

3. Ignoring the gravitational effects, write the dynamic model for this manipulator.

4. Write the dynamic model of this exercise in a form that is linear with respect to a set of dynamic parameters. In order to experimentally identify such parameters, which ones out of the position, velocity and acceleration of the two joint variables are needed?

¹The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2} \left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

EXERCISE 2

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta \mathbf{x} = \mathbf{\Phi} \Delta \zeta$$

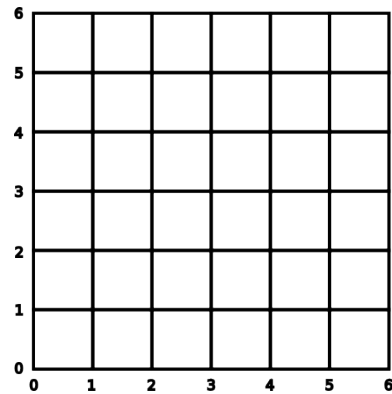
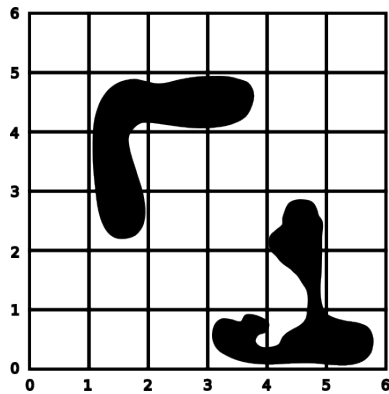
Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

3. Based on the equation previously commented, explain how the kinematic calibration algorithm works.
4. Discuss the concepts of repeatability and accuracy of the robot. With a good kinematic calibration do you expect to improve the repeatability or the accuracy?

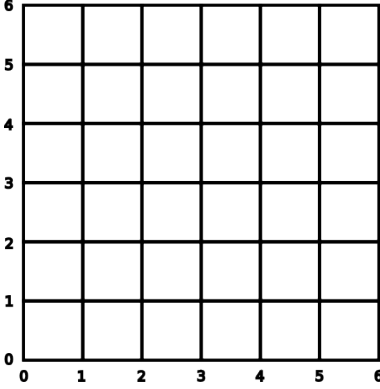
EXERCISE 3

1. Write the pseudocode for RRT planning algorithm.

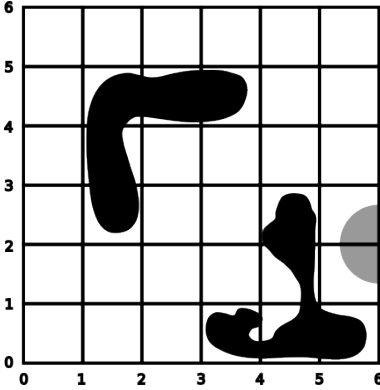
2. Consider the 2D environment depicted in the picture below, where the black areas represent obstacles. Using the grid in the picture below (right side), draw the corresponding occupancy grid.



3. Consider the occupancy grid previously determined. Randomly drawing from the configuration space gives rise to the following sequence $\mathbf{q}_1 = [3, 3]$, $\mathbf{q}_2 = [0, 6]$, $\mathbf{q}_3 = [0, 2]$, $\mathbf{q}_4 = [5, 4]$, $\mathbf{q}_5 = [2, 0]$, $\mathbf{q}_6 = [4, 6]$, $\mathbf{q}_7 = [6, 6]$, $\mathbf{q}_8 = [2, 6]$, $\mathbf{q}_9 = [6, 3]$, $\mathbf{q}_{10} = [6, 2]$. Using RRT, with $\mathbf{q}_{init} = [0, 0]$ and an exact steering function, construct the tree, and draw it in the picture below (if an edge touches the corner of a cell occupied by an obstacle there is no collision).



4. Consider the tree determined in the previous step and the picture below, where the grey area is the goal region. Starting from $\mathbf{q}_{init} = [0, 0]$ determine a path to the goal region, and draw it in the picture below.



EXERCISE 4

- Given the kinematic model of a mobile robot $\dot{\mathbf{q}} = G(\mathbf{q}) \mathbf{u}$ and its accessibility distribution Δ_A , write the conditions that allow to state if the model is STLA from $\mathbf{q}$ or STLC from $\mathbf{q}$.

2. Show that the unicycle model is STLC.

3. Consider the following two control laws

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{x_P} \begin{bmatrix} x_P \cos \theta - y_P \sin \theta & x_P \sin \theta + y_P \cos \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} v_{x_P} \\ v_{y_P} \end{bmatrix} \quad \begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{\varepsilon} \begin{bmatrix} \varepsilon \cos \theta & \varepsilon \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} v_{x_P} \\ v_{y_P} \end{bmatrix}$$

Answer the following questions:

- (a) at which robot model do the two controllers apply?
- (b) what is the model that describes the closed-loop system after the application of these control laws?
- (c) what are the main differences between the two control laws?

4. Consider a ROS architecture constituted by a control node and a simulator node. How should the *use_sim_time* parameter be set in order to test the control node using the simulator node? Clearly motivate the answer.