Control of Industrial and Mobile Robots

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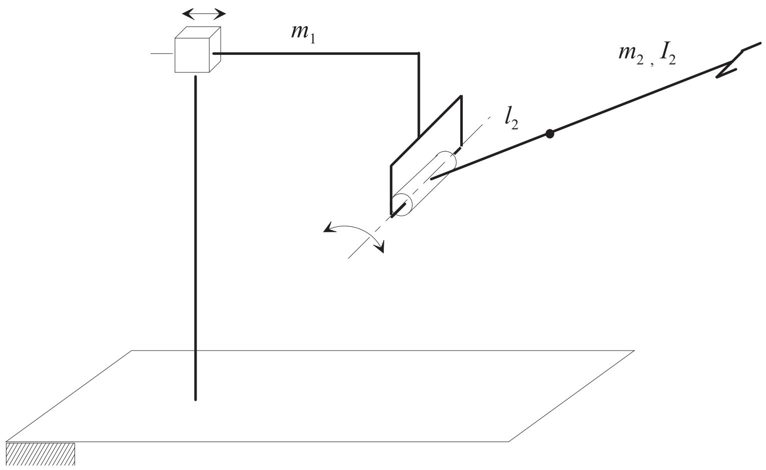
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1]

[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[2] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Ignoring the gravitational terms, write the equations of the dynamic model for this manipulator.

EXERCISE 2

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$\begin{array}{ccccc} t_1 = 0 & t_2 = 3 & t_3 = 5 & t_4 = 7 & t_5 = 10 \\ q_1 = 0 & q_2 = 45 & q_3 = 20 & q_4 = 50 & q_5 = 65 \end{array}$$

   Consider the interpolation of such points by a single polynomial of suitable degree. Write the vector equation that has to be solved for this specific example in order to find the coefficients of such polynomial.

2. What are the issues that suggest not to use the interpolation with a single polynomial?

3. Assume now that you want to use cubic polynomials in each interval. Assign suitable values to the velocity at the intermediate points.

4. If you use the spline method to interpolate the given points, which ones out of the position, the velocity and acceleration are continuous in all the intermediate time instants, i.e in the open interval (t_1, t_5)? And what about the initial and the final time instants t_1 and t_5?

EXERCISE 3

1. Are the following sentences true or false?

		T	F
(A)	Holonomic constraints reduce the available DOFs of a system	☐	☐
(B)	Holonomic constraints do not cause a loss of accessibility	☐	☐
(C)	Nonholonomic constraints do not locally limit the generalized velocities	☐	☐
(D)	Nonholonomic constraints do not cause any loss of accessibility	☐	☐

2. Consider the dynamic system representing the kinematic model of a robot, and assume it is not controllable. In this case, are the constraints representing the motion of the robot holonomic or nonholonomic? Clearly motivate the answer.

3. Consider the kinematic model of a bicycle with front-wheel drive, whose inputs are v and ω, and whose states are x, y, θ, and ϕ. Complete the following C++ function used by odeint to simulate the model, assuming that u1, and u2 are two variables of the class representing v, and ω, respectively.

```
void simulator::simulator_ode(const state_type &state, state_type &
    dstate, double t)
{







}
```

4. Assume that *state* is the topic used by the simulator to publish the state of the bicycle, and *state_publisher* is a variable of the node class, of type *ros::Publisher*, representing a publisher for that topic with type *std_msgs::Float64MultiArray*. Write the lines of code used by the node to publish the state $x = 5$, $y = 2$, $\theta = 0.2$, and $\phi = 0.1$.

EXERCISE 4

Consider the design of a trajectory tracking controller for a rear-wheel drive bicycle robot based on feedback linearization. Assume that the steering rate limit is so high that the steering angle can be changed instantaneously.

1. Given a point P located along the linear velocity vector, at a distance ε from the rear wheel contact point, derive the control laws of the feedback linearizing controller starting from the unicycle feedback linearization law.

2. Write the expression of a proportional trajectory tracking controller including velocity feed-forward, and draw the block diagram of the control system including the feedback linearizing controller and the trajectory tracking controller.

3. Determine the gains of the trajectory tracking controller in such a way that it has a bandwidth of 65 rad/s.

4. Prove that, even if the robot heading is no more controllable, it remains close to the heading of the reference trajectory. How can you modify the controller in order to have a robot heading, at the end of the reference trajectory, that is different from the reference trajectory heading?