Control of Industrial and Mobile Robots

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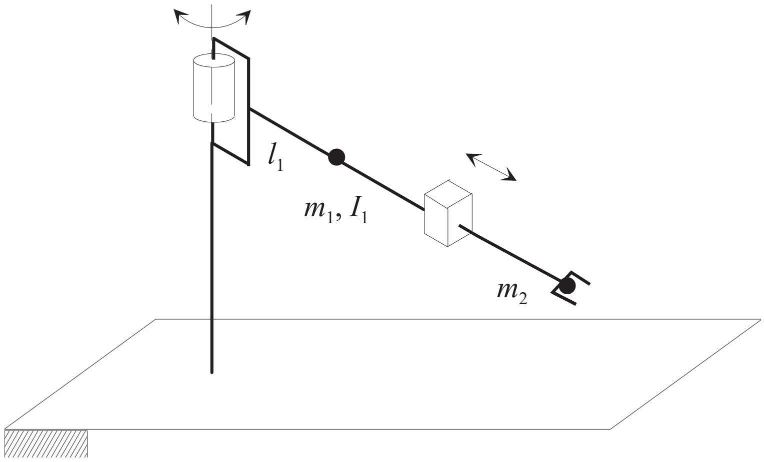
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator. Is this the only possible expression that matrix $\mathbf{C}$ can take?

3. Check that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator without gravity load, as the robot in this exercise, compute the expression of the derivative of the kinetic energy, exploiting the fact that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric. Specify whether this result can be obtained only with a matrix $\mathbf{C}$ computed from the Christoffel symbols or if it is general.

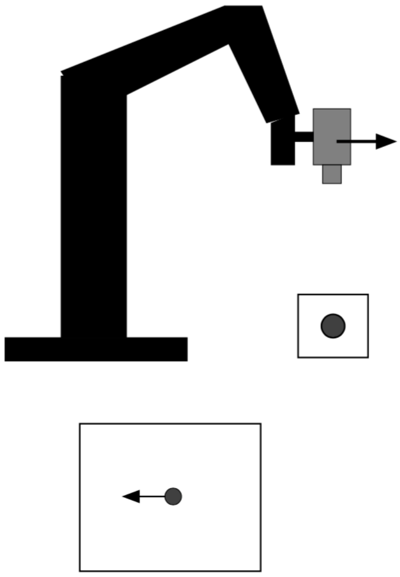
EXERCISE 2

Consider the control of a manipulator with vision sensors.

1. Explain what are the "eye-in-hand" and the "eye-to-hand" configurations, mentioning some pros and cons of both solutions.

2. Making reference to the following picture, where a single image point is considered, explain what is the interaction matrix in the context of visual control, specifying precisely:

   - the variables that are related by the interaction matrix
   - the size of the interaction matrix
   - the variables upon which the interaction matrix depends
   - which columns of the matrix depend on the depth Z



3. Explain what is the image Jacobian, what is the size of such matrix, and what is its relation with the interaction matrix.

4. Sketch the block diagram of a look-and-move, image-based, vision control system and specify the expression of a control law based on the image Jacobian.

EXERCISE 3

1. Consider a system of kinematic constraints in Pfaffian form $A^T(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}$ where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$ and

$$A^T(\mathbf{q}) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & q_1 & 1 & 0 \end{bmatrix}$$

Assuming that two vectors in the null space of $A^T(\mathbf{q})$ are

$$\mathbf{g}_1(\mathbf{q}) = \begin{bmatrix} 0 & 1 & -q_1 & 0 \end{bmatrix}^T \qquad \mathbf{g}_2(\mathbf{q}) = \begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}^T$$

write the kinematic model of the robot whose motion is described by the two kinematic constraints.

2. Referring to the kinematic constraints of the previous question, write the vector fields describing all possible motions of the robot, and determine if all points of the configuration space are accessible by the robot.

3. Consider another robot, moving in the xy-plane and whose configuration is described by $\mathbf{q} = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$. The motion of the robot is characterised by the constraint $\dot{x}_b = 0$, where (x_b, y_b) is the local robot frame. Write the kinematic constraint, in Pfaffian form, at which the robot is subjected.

4. Consider the following kinematic model

$$
\begin{aligned}
\dot{x} &= -v \cos\phi \cos\theta \\
\dot{y} &= -v \sin\phi \cos\theta \\
\dot{z} &= v \sin\theta \\
\dot{\theta} &= \omega_1 \\
\dot{\phi} &= \omega_2
\end{aligned}
$$

Complete the following C++ function used by odeint to simulate the model, assuming that u1, u2, u3, are three variables of the class representing v, ω_1, and ω_2, respectively.

```
void simulator::simulator_ode(const state_type &state, state_type &
    dstate, double t)
{




}
```

EXERCISE 4

1. In an optimal sampling-based planning algorithm what are the two assumptions the cost function should satisfy? What is the additional property a cost should satisfy in RRT*?

2. Write the pseudocode of the rewiring function of RRT* planner.

3. Consider kinodynamic RRT*, assuming a unicycle kinematic model for the robot and a configuration space described by $\{x, y, \theta\}$. Write the analytical formulation of the two point boundary value problem (exact steering) that must be solved in order to connect two nodes $\mathbf{q}_i$ and $\mathbf{q}_{i+1}$ in minimum time, but weighting the control effort. Include also the two actuation constraints $v \in [v_{min}, v_{max}]$ and $\omega \in [\omega_{min}, \omega_{max}]$.

4. What are the three main groups of kinodynamic constraints considered by a kinodynamic sampling-based planner? Provide an example for each group.