Control of Industrial and Mobile Robots

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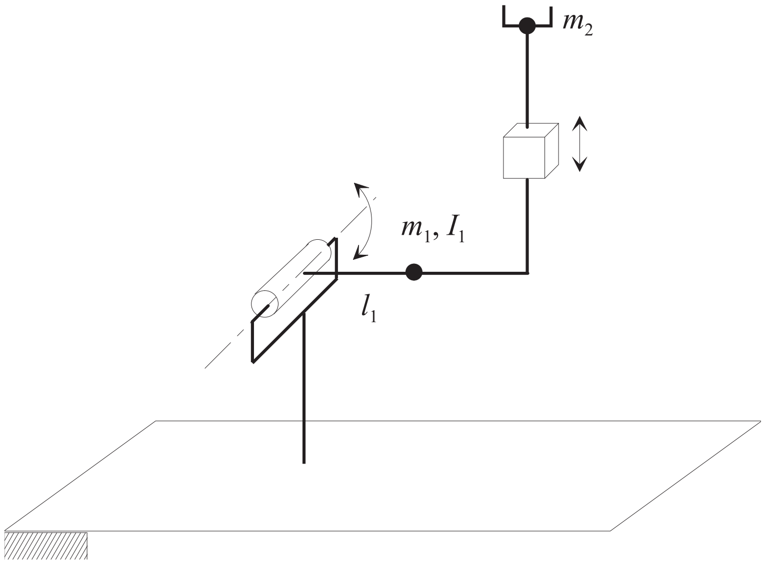
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Consider the adoption of an inverse dynamics controller for this robot: ignoring the gravitational terms, write the equations of such controller.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. In order to make the control system stable even if the knowledge of the model of the robot is only partial, what kind of control law can be used? What is a possible issue with use of such control law?

EXERCISE 2

1. Explain why for the kinematic scaling of trajectories it is possible to consider each joint separately, while for the dynamic scaling this is not possible.

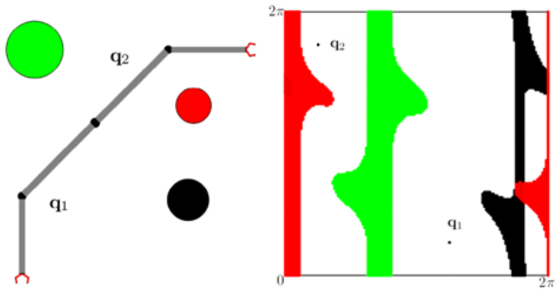
2. Consider the following equation:

$$\tau_i(t) = \alpha_i\left(r(t)\right)\ddot{r}(t) + \beta_i\left(r(t)\right)\dot{r}^2(t) + \gamma_i\left(r(t)\right)$$

   Explain whether such equation is used in the kinematic scaling or in the dynamic scaling and define all symbols used in the equation.

3. Assume now that, in a robot that is not affected by gravity, trajectories have been planned such that the torque of one joint exceeds its limit by 21% (the torques of the other joints are within their limits). Explain how the trajectory can be scaled and what should be the scaling factor.

4. With specific reference to the following picture, define the concept of "configuration space" in the path planning with obstacle avoidance problem.



EXERCISE 3

1. Given the kinematic model $\dot{\mathbf{q}} = G(\mathbf{q})\,\mathbf{u}$, where $\mathbf{q} \in \mathbb{R}^n$, derived from k kinematic constraints, answer to the following questions, clearly motivating each answer:
   - How many columns does matrix $G(\mathbf{q})$ consist of?
   - How are these columns derived from the kinematic constraints?

2. Calling α the dimension of the accessible configuration space, answer to the following questions, clearly motivating each answer:
   - For which values of α the system is controllable?
   - If the system is not controllable, for which values of α the set of constraints is nonholonomic and for which values holonomic?

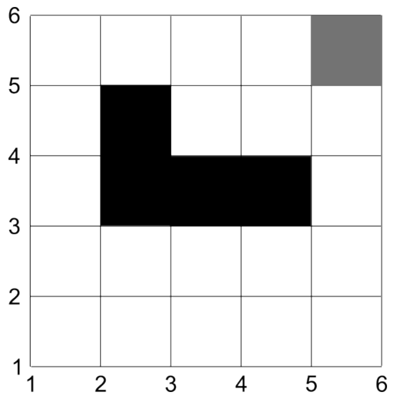
3. Consider now the unicycle kinematic model. Determine the analytical expression of the vector fields that represent a base of the accessible space and the dimension of the accessible space.

4. Considering again the unicycle kinematic model, provide a physical interpretation for the vector fields that represent a base of the accessible space.

EXERCISE 4

1. Write the pseudocode of RRT planning algorithm.

2. Consider the 2D environment depicted in the figure below, where the black rectangles are obstacles, the gray square is the goal region, and $\mathbf{q}_s = [1,1]$ is the initial node.



Randomly drawing from the configuration space gives rise to the following sequence $\mathbf{q}_1 = [4,5]$, $\mathbf{q}_2 = [3,2]$, $\mathbf{q}_3 = [6,5]$, $\mathbf{q}_4 = [1,3]$, $\mathbf{q}_5 = [5,2]$, $\mathbf{q}_6 = [6,3]$, $\mathbf{q}_7 = [3,6]$, $\mathbf{q}_8 = [6,1]$, $\mathbf{q}_9 = [2,6]$, $\mathbf{q}_{10} = [6,6]$.

Using RRT with an exact steering function, draw the tree and write the set of nodes Q computed by the algorithm.

3. Using the three drawn in the previous step, find a path starting from $\mathbf{q}_s$ and ending in the goal region, if one exists. Write the sequence of nodes representing the path and compute its length.

4. Applying RRG to the sequence of nodes reported in step 2, using constant radius for the near set equal to 2. Which edges are added to the tree determined in that step?