# Control of Industrial and Mobile Robots 

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## Warnings

- This file consists of $\mathbf{1 0}$ pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given either in English or in Italian.
- Solutions and answers must be given exclusively in the reserved space. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to hand this file only. Every other sheet you may hand will not be taken into consideration.


## EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:


Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.
2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms ${ }^{1}$ for this manipulator.
3. Write the complete dynamic model for this manipulator.

[^0]4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters.

## EXERCISE 2

1. Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:


Express the natural and the artificial constraints for this problem, and specify the selection matrix.
2. Sketch the block diagram of a hybrid force-position controller. What are possible sources of inconsistency in the adoption of such scheme?
3. Explain what an implicit force controller is and why it might be convenient with respect to an explicit solution.
4. Suppose now that along the translational $z$ direction an implicit force controller has to be designed. Sketch the block diagram of such controller and design it taking a bandwidth of $20 \mathrm{rad} / \mathrm{s}$.

## EXERCISE 3

1. Consider a unicycle mobile robot. Selecting as flat outputs $z_{1}=x$ and $z_{2}=y$, write the flat model of the robot, i.e., the analytical relations from $z_{1}, z_{2}$ to $x, y, \theta$ and from $z_{1}, z_{2}$ to $v, \omega$.
2. Using the flatness transformation, determine the analytic expression of a trajectory $x(t), y(t)$ (and the numerical values of its coefficients) that moves a unicycle robot, in an obstacle free environment, from an initial state $x_{i}=y_{i}=\theta_{i}=0$ and $v_{i}=0$ at $t_{i}=0$, to a final state $x_{f}=y_{f}=5, \theta_{f}=0$ and $v_{f}=0$ at $t_{f}=1$.
3. Modify the answer to the previous step in order to introduce the minimization of the cost

$$
J(v, \omega)=\int_{0}^{T_{f}}\left(v^{2}+0.1 \omega^{2}\right) \mathrm{d} t
$$

where now $T_{f}$ is a free parameter. Write the analytical expression of the relations that allow to compute the additional coefficients that must be introduced in order to enforce the minimization of the cost function.
4. Consider now an environment with obstacles, where each obstacle can be represented by a circle of radius $R_{i}$ and center $\left(c_{x_{i}}, c_{y_{i}}\right)$. Write the constraint that must be included in the optimization problem considered in the previous step, in order to guarantee obstacle avoidance.

## EXERCISE 4

Consider the design of a trajectory tracking controller for a unicycle robot based on feedback linearization.

1. Write the analytical relations that define the coordinate transformation from the unicycle wheel contact point to point $P$, i.e., the new reference point considered to solve the trajectory tracking problem.
2. Starting from the coordinate transformation in step 1, derive the control laws of the feedback linearizing controller.
3. Using the control laws derived in the previous step and the unicycle kinematic model, derive the expression of the dynamic system representing the closed-loop system obtained connecting the linearizing controller and the kinematic model.
4. Draw the block diagram of the complete trajectory tracking controller, including the feedback linearizing controller, the robot model, and the trajectory tracking controller. Write the equations of the dynamic system that must be used in order to design the trajectory tracking controller.

[^0]:    ${ }^{1}$ The general expression of the Christoffel symbols is $c_{i j k}=\frac{1}{2}\left(\frac{\partial b_{i j}}{\partial q_{k}}+\frac{\partial b_{i k}}{\partial q_{j}}-\frac{\partial b_{j k}}{\partial q_{i}}\right)$

