Control of Industrial and Mobile Robots

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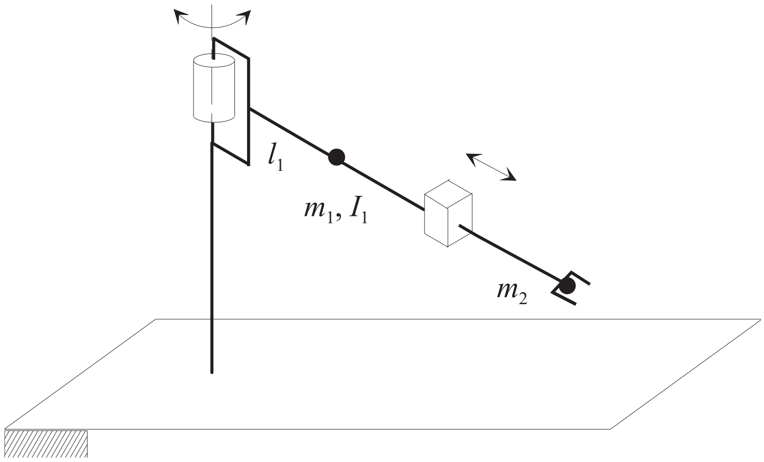
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

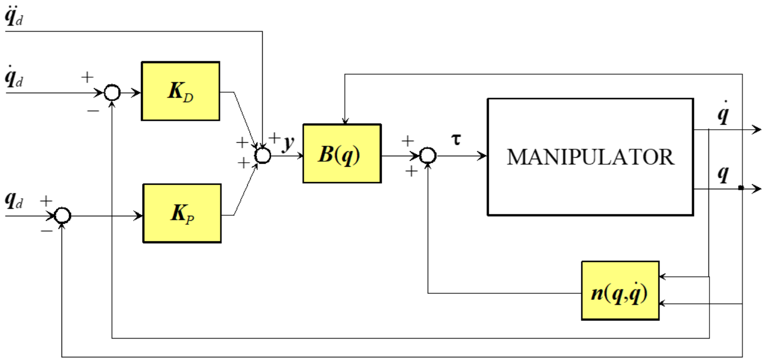
3. Check that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator, compute the expression of the derivative of the kinetic energy, exploiting the fact that matrix $\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

EXERCISE 2

1. Consider the control scheme sketched in the following picture:



Explain which control scheme it refers to and what is the result in terms of closed-loop dynamics that can be achieved with such control scheme.

2. In a two-link planar manipulator in the vertical plane with prismatic joints, the inertia matrix and the gravitational terms take the following expressions, respectively:

$$\begin{aligned}\mathbf{B} &= \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \\ \mathbf{g} &= \begin{bmatrix} (m_1 + m_2)\, g \\ 0 \end{bmatrix}\end{aligned}$$

Write the expression (equation by equation) of the control law for the control scheme of this exercise, specific for this manipulator.

3. Tune the two matrices $\mathbf{K}_P$ and $\mathbf{K}_D$ in such a way that the dynamics of the error in the two joints is identical with two real eigenvalues at frequency 20 rad/s

4. Suppose now that there is uncertainty in the dynamic model of the robot. When is it appropriate to use a robust control system and when is it to use an adaptive one? Which one is a switching control law and why can this be a problem?

EXERCISE 3

1. Consider the following kinematic constraint

$$2x^2\dot{y} - 4(x-1)\dot{z} + 3\dot{w} = 0$$

   where $\mathbf{q} = \begin{bmatrix} x & y & z & w \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Consider the following kinematic constraint

$$3\dot{x} - (x-1)\dot{y} + 2\dot{z} = 0$$

   Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Consider the following system of kinematic constraints in Pfaffian form

$$A^T(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}$$

where $A^T(\mathbf{q})$ is a 2×4 matrix and $\mathrm{rank}\left(A^T(\mathbf{q})\right) = 2$.
Describe the analytical procedure to verify if the system of constraints is holonomic or nonholonomic.

4. Consider a mobile robot whose configuration is represented by $\mathbf{q} \in \mathbb{R}^4$, and whose motion is described by the system of kinematic constraints of the previous steps. Show that the following kinematic model

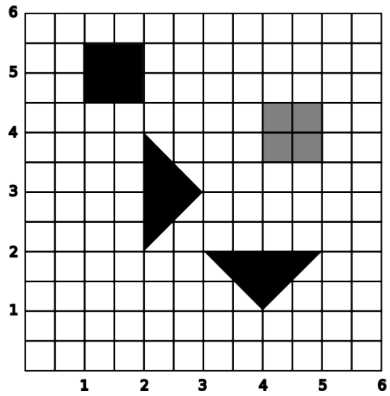
$$\dot{\mathbf{q}} = \begin{bmatrix} 2(2x-1) \\ 6(1-x) \\ -3x^2 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} x-1 \\ 3 \\ 0 \\ -2x^2 \end{bmatrix} u_2$$

describes the motion of the robot.

EXERCISE 4

1. Write the pseudocode of RRT planning algorithm.

2. Consider the 2D environment depicted in the figure below



where the black square and the black triangles are obstacles, while the gray square is the goal region. Using RRT draw the tree and find a path starting from $\mathbf{q}_1 = [1, 1]$, using the following randomly sampled configurations $\mathbf{q}_2 = [5.5, 1]$ $\mathbf{q}_3 = [2, 0.5]$ $\mathbf{q}_4 = [2.5, 2]$ $\mathbf{q}_5 = [3.5, 0.5]$ $\mathbf{q}_6 = [3.5, 3]$ $\mathbf{q}_7 = [0.5, 3.5]$ $\mathbf{q}_8 = [4.5, 3.5]$ $\mathbf{q}_9 = [1.5, 3.5]$ $\mathbf{q}_{10} = [3, 5]$.
Note that, tree edges can be tangent to obstacles or pass through an obstacle vertex, and an exact steering function is considered.

3. Considering a disk robot with a diameter of 0.5, draw the environment modifying the obstacles in order to take the footprint of the robot into account.

4. Write the pseudocode of RRG and explain the main differences with respect to RRT.