Control of Industrial and Mobile Robots

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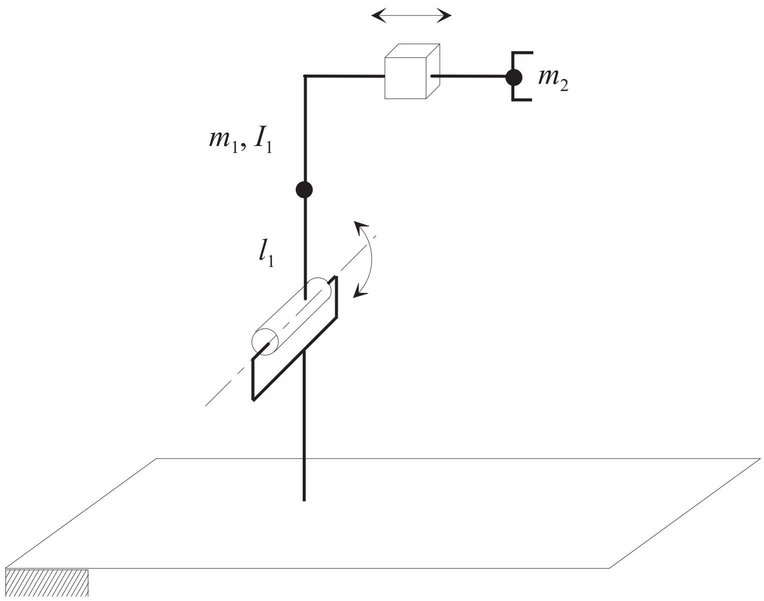
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

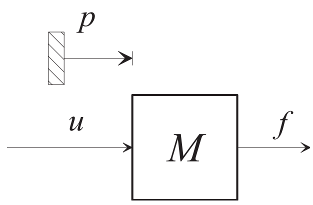
3. Ignoring the gravitational terms, write the complete dynamic model for this manipulator.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For this specific manipulator, wtite the expression of the kinetic energy. Is it possible that this kinetic energy is zero for joint velocities different from zero?

EXERCISE 2

1. Consider a single mass affected by an external force f and a control force u:



Write the expression of an impedance control law that makes the system react to the external force f like a mass-spring-damper system, with all parameters assignable.

2. Consider a manipulator where a system of forces is applied at the end-effector. Discuss the statics of the manipulator, i.e. find analytically the relation between this system of force and the joint torques at the equilibrium.

3. Write the expressions of a translational impedance and the expression of a rotational impedance.

4. Assume now that a force sensor at the end effector is unavailable. Discuss a method to estimate force and moments at the end effector, making reference to the concept of residual vector.

EXERCISE 3

1. Consider the following kinematic constraint

$$x^2\dot{y} + (1 - x)\dot{z} + \dot{w} = 0$$

   where $\mathbf{q} = \begin{bmatrix} x & y & z & w \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Consider the following kinematic constraint

$$6\dot{x} + (1 - x)\dot{y} + 4\dot{z} = 0$$

   Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Consider the following system of kinematic constraints in Pfaffian form

$$A^T(\mathbf{q})\,\dot{\mathbf{q}} = \begin{bmatrix} 0 & x^2 & 1-x & 1 \\ 6 & 1-x & 4 & 0 \end{bmatrix} \dot{\mathbf{q}} = \mathbf{0}$$

Determine if the system of constraints is holonomic or nonholonomic.
Note that, a basis of the null space of $A^T(\mathbf{q})$ is composed by the two vectors

$$g_1(\mathbf{q}) = \begin{bmatrix} 0 \\ -4 \\ 1-x \\ 3x^2+2x-1 \end{bmatrix} \qquad g_2(\mathbf{q}) = \begin{bmatrix} -4 \\ 0 \\ 6 \\ 6(x-1) \end{bmatrix}$$

4. Consider a mobile robot whose configuration is represented by $\mathbf{q} \in \mathbb{R}^4$, and whose motion is described by the system of kinematic constraints of the previous step. Show that the following kinematic model

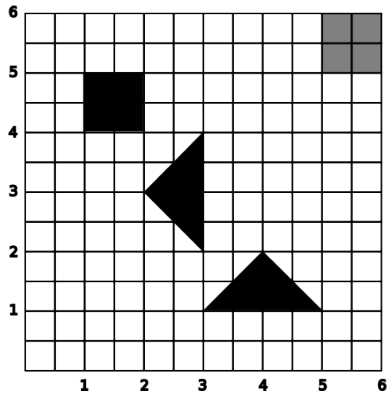
$$\dot{\mathbf{q}} = \begin{bmatrix} 0 \\ -4 \\ 1-x \\ 3x^2+2x-1 \end{bmatrix} u_1 + \begin{bmatrix} -4 \\ 0 \\ 6 \\ 6\,(x-1) \end{bmatrix} u_2$$

describes the motion of the robot.

EXERCISE 4

1. Write the pseudocode of RRT planning algorithm.

2. Consider the 2D environment depicted in the figure below



where the black square and the black triangles are obstacles, while the gray square is the goal region. Using RRT draw the tree and find a path starting from $\mathbf{q}_1 = [1, 1]$, using the following randomly sampled configurations $\mathbf{q}_2 = [5, 4]$ $\mathbf{q}_3 = [2, 3.5]$ $\mathbf{q}_4 = [2.5, 2]$ $\mathbf{q}_5 = [3, 5]$ $\mathbf{q}_6 = [4, 4]$ $\mathbf{q}_7 = [0.5, 3.5]$ $\mathbf{q}_8 = [5.5, 3]$ $\mathbf{q}_9 = [2, 0.5]$ $\mathbf{q}_{10} = [5.5, 5.5]$. Note that, tree edges can be tangent to obstacles or pass through an obstacle vertex, and an exact steering function is considered.

3. Considering a disk robot with a diameter of 0.5, draw the environment modifying the obstacles in order to take the footprint of the robot into account.

4. Write the pseudocode of RRG and explain the main differences with respect to RRT.