Control of Industrial and Mobile Robots

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NAME:
UNIVERSITY ID NUMBER:

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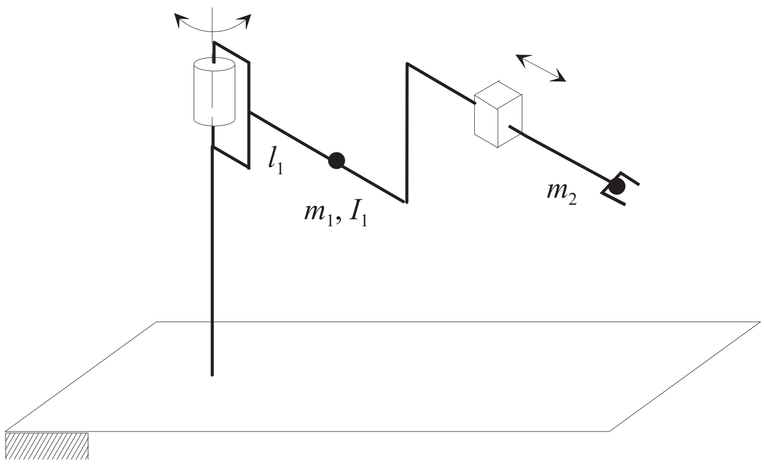
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the complete dynamic model for this manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters. Is it possible to identify the mass of the first link with experiments based on such model?

EXERCISE 2

Consider a kinematically redundant manipulator.

1. Write the general expression of the solutions of the inverse kinematics problem at velocity level, specifying what is the projection matrix and what is its role. Write also the cost function whose minimization leads to this general solution.

2. Express the solution in the form that includes a closed loop correction (kinematic control) and explain why this correction is used.

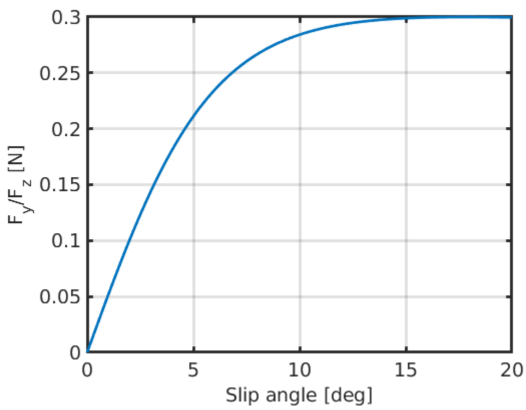
3. Discuss some criteria to select the joint velocities $\dot{\mathbf{q}}_0$ to be projected onto the null space of the Jacobian matrix.

4. Consider a robot with seven joints and a task that concerns only the position of the TCP. What is the size of the vector $\dot{\mathbf{q}}_0$ of joint velocities to be projected onto the null space of the Jacobian matrix and what is the size of the vector after such projection?

EXERCISE 3

1. As a result of an experimental campaign performed on snow, a tire lateral force has been characterised interpolating experimental data with the following Pacejka Magic Formula (whose plot is shown in the picture below).

$$\frac{F_y}{F_z} = 0.3 \sin\left(2 \arctan\left(5\alpha - \left(5\alpha - \arctan\left(5\alpha\right)\right)\right)\right)$$



Determine, using the Magic Formula and motivating the result, the value of the cornering stiffness, and draw on the picture the cornering stiffness approximation of the lateral force/slip relation.

2. The cornering stiffness approximation cannot represent the tire force saturation. Illustrate a modelling approach (different from the Pacejka Magica Formula), writing the expression of the force-slip relation, that allows to represent the saturation and is suitable for model-based control.

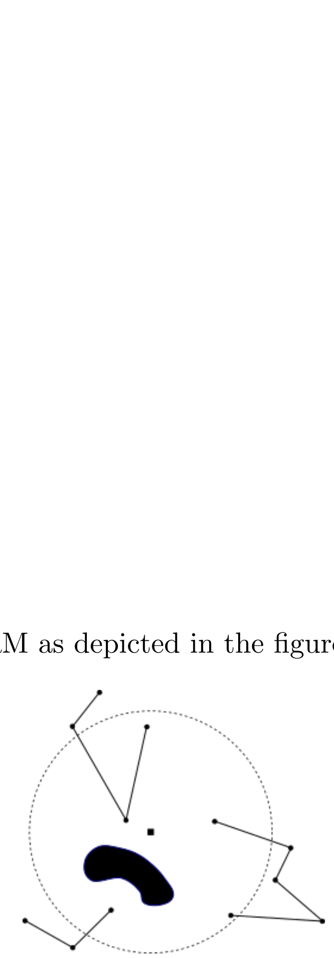
3. During a curve the same tire is characterised by a slip angle of 5 deg. What is the corresponding value of the lateral force F_y, assuming $F_z = 150$ N? What is the maximum longitudinal force F_x the tire can generate in these conditions?

4. Clearly explain what are the most suitable lateral force models for model-based design and simulation, and when it is appropriate to use a lateral force model including the saturation effect.

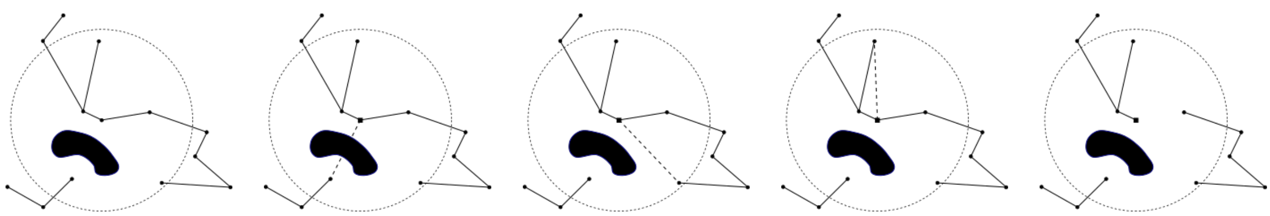
EXERCISE 4

1. Write and explain the pseudocode of the algorithm that allows to construct the probabilistic roadmap used by PRM.

2. Consider the i-th iteration of PRM as depicted in the figure below



where the black square is the current $\mathbf{q}_{rand}$, the black blob region is an obstacle, the black dashed circle centred in $\mathbf{q}_{rand}$ is the region defining the Near set, and black dots and segments are nodes and edges in the actual V and E sets, respectively.
Put the following pictures in the correct order, according to the execution of the algorithm across the nodes belonging to the Near set (black dashed edges represent connection attempts that are discarded).



A B C D E

3. How could the path resulting from PRM be used to speed up the computation of RRT*? Clearly motivate the answer.

4. Consider again an RRT* planner, what are the characteristics a cost function should satisfy in order to be included into the planner?