Control of Industrial and Mobile Robots

Prof. Rocco, Bascetta

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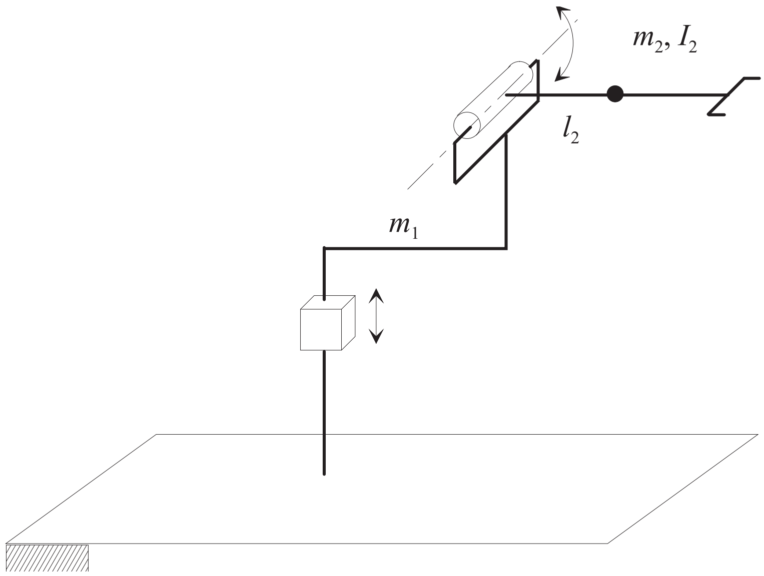
NAME:
UNIVERSITY ID NUMBER:
SIGNATURE: ______________________________

Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

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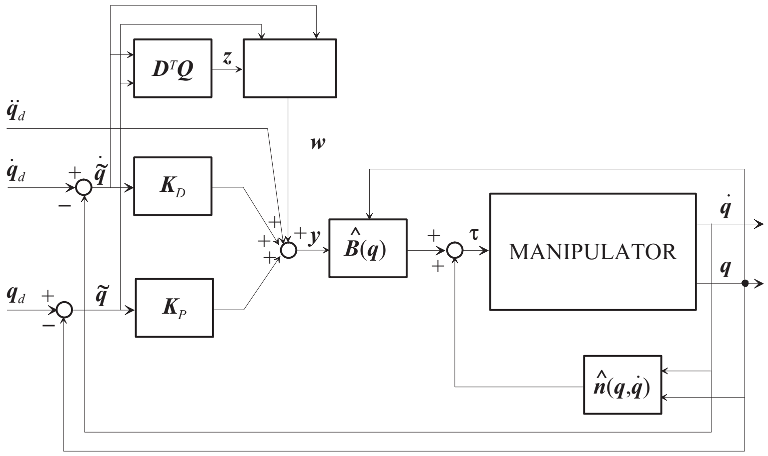
[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the gravitational terms for this robot.

3. Write the expression of an inverse dynamics controller in joint space for this specific manipulator that ignores centrifugal and Coriolis terms.

4. If you want to make the control system robust against a partial knowledge of the dynamic model of the robot (for example lack of knowledge of the centrifugal and Coriolis terms) you can use a particular control law. Making reference to the following sketch, briefly comment this control law and explain the nature of the additional term computed in the empty block of the sketch.



EXERCISE 2

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$
\begin{array}{ccccc}
t_1 = 0 & t_2 = 1 & t_3 = 5 & t_4 = 7 & t_5 = 10 \\
q_1 = 10 & q_2 = 0 & q_3 = 30 & q_4 = 40 & q_5 = 55
\end{array}
$$

Assign suitable values to the speed at the intermediate points.

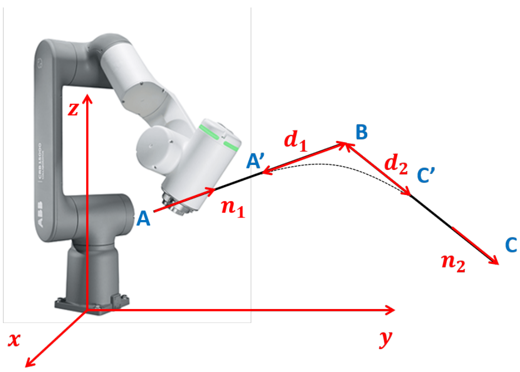
2. Using the values of speed previously evaluated, compute the expression of the cubic polynomial for the first interval (from t_1 to t_2).

3. In the spline method, the following equation has to be solved:

$$\mathbf{A}\mathbf{v} = \mathbf{c}$$

Explain what is the meaning of $\mathbf{v}$, what is the size of matrix $\mathbf{A}$ and whether matrix $\mathbf{A}$ has any particular shape.

4. Consider now the concatenation of linear paths. Making reference to the following picture, and without going through the mathematics, explain what are the assumptions that are enforced to compute the blending:



EXERCISE 3

1. With reference to an optimal planning problem, which of the following functions can be used as cost function:

   (a) trajectory duration;

   (b) path length;

   (c) electrical energy consumption, for a robot that cannot recharge the battery during the execution of the path;

   (d) electrical energy consumption, for a robot that can recharge the battery during the execution of the path.

   Motivate the answer.

2. With reference to RRT* optimal planner, explain the rewire procedure and, using an example, show how the rewire procedure modifies the tree.

3. With reference to kinodynamic RRT* planner applied to the kinematic model of a unicycle robot, with the aim of minimizing the duration of the trajectory while penalizing the control effort, write the problem that must be solved to compute an edge of the tree (exact optimal steering problem).

4. With reference to a feasible planning problem, write the definition of the steering function (non-exact steering).

EXERCISE 4

1. The motion of a robot can be represented using the bicycle kinematic model, with $0 \leq v \leq v_M$ and $\phi_m \leq \phi \leq \phi_M$, and assuming the steering angle can be changed instantaneously. To design the trajectory tracking controller we adopt the transformation to the canonical simplified model, the feedback linearization, and a PI position controller.
   Write the relations that allow to transform the bicycle kinematic model into the canonical simplified model, and the actuation constraints at which the canonical simplified model is subjected.

2. Considering a point P at a distance ε from the wheel contact point, write the equations of the feedback linearizing controller that allows to linearize the canonical simplified model.

3. Write the analytical equations of the complete trajectory tracking controller, that allow to compute v and ϕ given the desired trajectory $x^d(t)$, $y^d(t)$, $\theta^d(t)$, and a measure of the robot position and heading, $x(t)$, $y(t)$, $\theta(t)$. The position controller is a PI controller.

4. Draw the block diagram of the complete trajectory tracking controller.