Control of Industrial and Mobile Robots

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NAME:
UNIVERSITY ID NUMBER:

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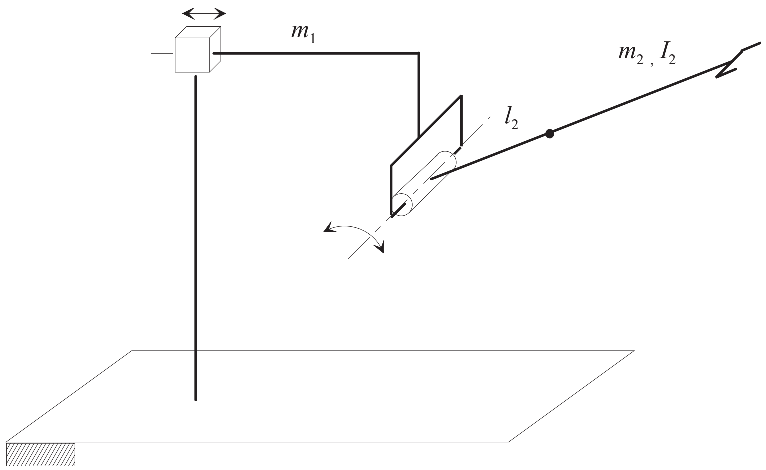
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

Consider the manipulator sketched in the picture:



1. Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1]

[1] The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

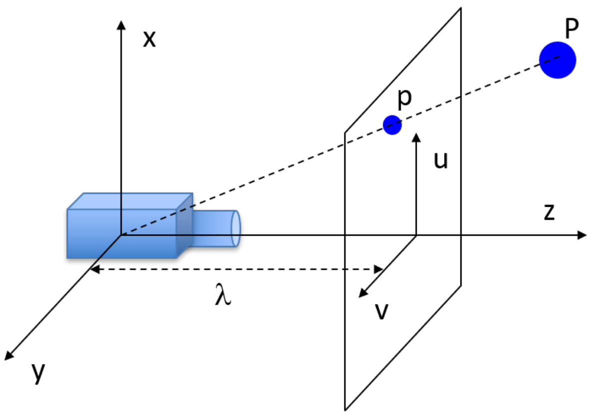
4. For a generic manipulator, ignoring the gravitational terms and exploiting the skew symmetry of matrix $\mathbf{N}$, obtain an expression of the derivative with respect to time of the kinetic energy.

[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

EXERCISE 2 Consider a robot that uses a camera.

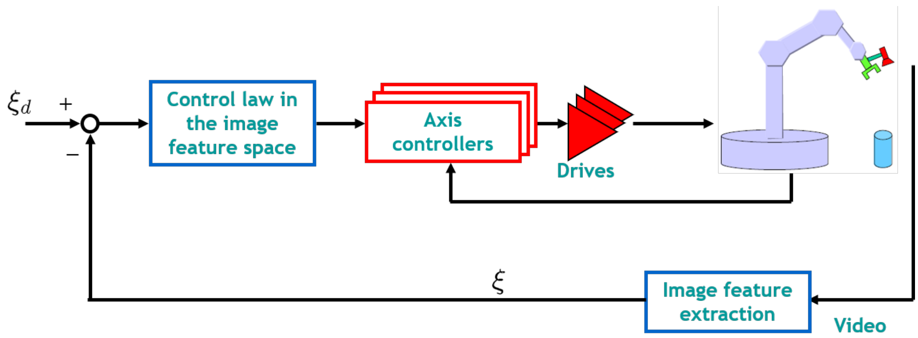
1. Explain what are the extrinsic and the intrinsic calibrations, making in particular reference to the notion of camera intrinsic matrix.

2. With reference to the following sketch, define what an image feature is and write the equations of the perspective projection method.



3. Define the interaction matrix and the image Jacobian for a vision-based robotic system, in terms of the quantities that each of the two matrices relate.

4. Consider now the following block diagram:



Is this a look-and-move or a visual servoing scheme? A position-based or an image-based scheme? Write an expression of the control law that can be used in this control scheme.

EXERCISE 3

1. Given the kinematic constraint

$$\dot{q}_1 + \dot{q}_4 = 0$$

   where $\mathbf{q} \in \mathbb{R}^4$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Given the kinematic constraint

$$q_1 \dot{q}_2 + \dot{q}_3 = 0$$

   where $\mathbf{q} \in \mathbb{R}^4$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Is the system of constraints

$$\dot{q}_1 + \dot{q}_4 = 0$$
$$q_1\dot{q}_2 + \dot{q}_3 = 0$$

holonomic or nonholonomic? Motivate the answer analysing the accessibility distribution. Note that rank $\left(A^T\left(\mathbf{q}\right)\right) = 2$, and two vectors in the null space of $A^T\left(\mathbf{q}\right)$ are

$$g_1\left(\mathbf{q}\right) = \begin{bmatrix} 0 \\ 1 \\ -q_1 \\ 0 \end{bmatrix} \qquad g_2\left(\mathbf{q}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

4. Consider a mobile robot, whose configuration is represented by $\mathbf{q} \in \mathbb{R}^4$, and whose motion is described by the set of constraints introduced in the previous question.
Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} 0 \\ 1 \\ -q_1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} u_2$$

describe the motion of the robot? Clearly motivate the answer.

EXERCISE 4

1. The trajectory tracking controller for a robot, modelled using the bicycle kinematic model, is designed exploiting the canonical simplified model. Write the relation that allows to transform the bicycle kinematic model into the canonical simplified model.
Under which assumptions the transformation can be applied?

2. The robot is characterised by the following actuation constraints $0 \leq v \leq v_M$ and $\phi_m \leq \phi \leq \phi_M$. Show how these constraints can be remapped into constraints on the control variables of the canonical simplified model.

3. Draw the block diagram of the complete trajectory tracking controller based on the canonical simplified model transformation, and write the equations representing each block.

4. A more simple control solution can be devised transforming the bicycle into the canonical simplified model, and then adopting an open-loop control strategy based on the flatness transformation. Write (and explain) the equations of the trajectory tracking controller, assuming that an analytical expression of the desired trajectory, as two functions of time, $x^d(t)$, $y^d(t)$, is available to the controller.