# Control of Industrial and Mobile Robots

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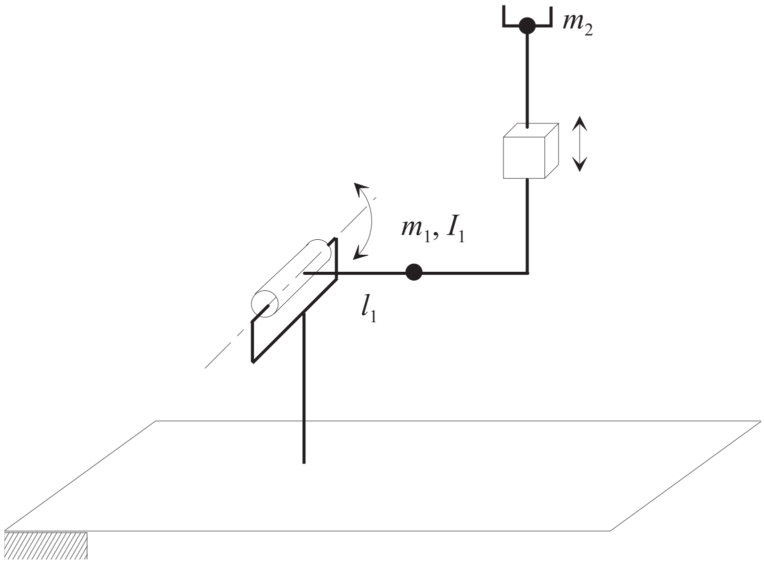
**Warnings**

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

**EXERCISE 1**

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:




Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Check that the expression $\dot{\mathbf{q}}^T \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = 0$ , where $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, is true in this case.

4. Disregarding the gravitational terms, write the dynamic model of this manipulator in a form that is linear with respect to a set of dynamic parameters.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

**EXERCISE 2**

1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$\begin{array}{ccccc} t_1 = 0 & t_2 = 2 & t_3 = 5 & t_4 = 8 & t_5 = 10 \\ q_1 = 0 & q_2 = -5 & q_3 = 10 & q_4 = 40 & q_5 = 60 \end{array}$$

   Assign suitable values to the speed at the intermediate points.

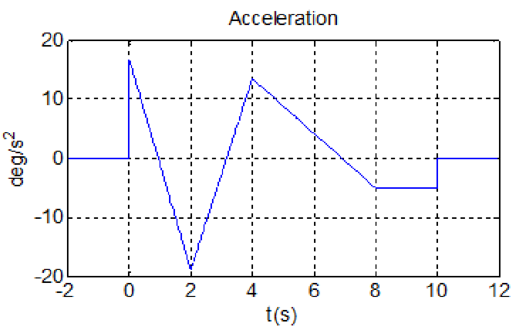
2. Using the values of speed previously evaluated, compute the expression of the cubic polynomial for the first interval (from $t_1$ to $t_2$).

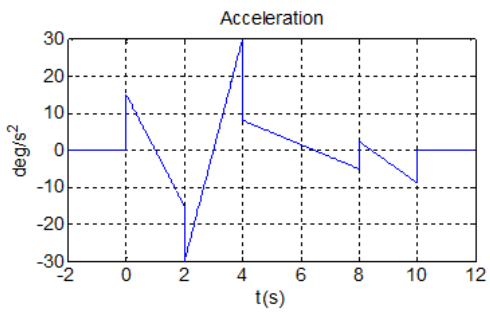
3. In the spline method, the following equation has to be solved:

$$\mathbf{A}\mathbf{v} = \mathbf{c}$$

Explain what is the meaning of the symbols used in this equation, what are their sizes, and whether matrix $\mathbf{A}$ has any particular shape.

4. Explain which one of the acceleration profiles shown in the following pictures has been obtained with the spline method:



**EXERCISE 3**

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\,\dot{x} + Y(\mathbf{q})\,\dot{y} + Z(\mathbf{q})\,\dot{z} = 0$$

   where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 - 2q_1\dot{q}_2 + 5\dot{q}_3 = 0 \qquad 7q_1\dot{q}_1 + q_2^2\dot{q}_3 = 0$$

   where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Determine, using the necessary and sufficient condition, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider two mobile robots, whose configurations are described by $\mathbf{q} \in \mathbb{R}^3$, and whose motion is described by $\dot{q}_1 - 2q_1\dot{q}_2 + 5\dot{q}_3 = 0$ for the first robot, and $7q_1\dot{q}_1 + q_2^2\dot{q}_3 = 0$ for the second robot. Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} 2q_1 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} u_2$$

describe the motion of the first robot, and the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} -q_2^2 \\ 0 \\ 7q_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

the motion of the second robot?
Clearly motivate the answer.

4. Considering the second constraint ($a^T(\mathbf{q})\dot{\mathbf{q}} = 7q_1\dot{q}_1 + q_2^2\dot{q}_3 = 0$), two vectors in the null of $a^T(\mathbf{q})$ are

$$\mathbf{g}_1 = \begin{bmatrix} -q_2^2 \\ 0 \\ 7q_1 \end{bmatrix} \qquad \mathbf{g}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Compute the vector field representing the motion that is locally constrained by $7q_1\dot{q}_1 + q_2^2\dot{q}_3 = 0$.

**EXERCISE 4**

1. Write the definitions of *small-time local accessibility* and *small-time local controllability*.

2. Write the conditions that must be checked to verify if the kinematic model of a mobile robot is small-time locally accessible or small-time locally controllable.

3. Using an example, provide an intuitive interpretation of all the conditions introduced in the previous step.

4. Show how, applying the conditions introduced in the previous step, one can determine if the canonical simplified model is small-time locally accessible or small-time locally controllable.