Control of Industrial and Mobile Robots

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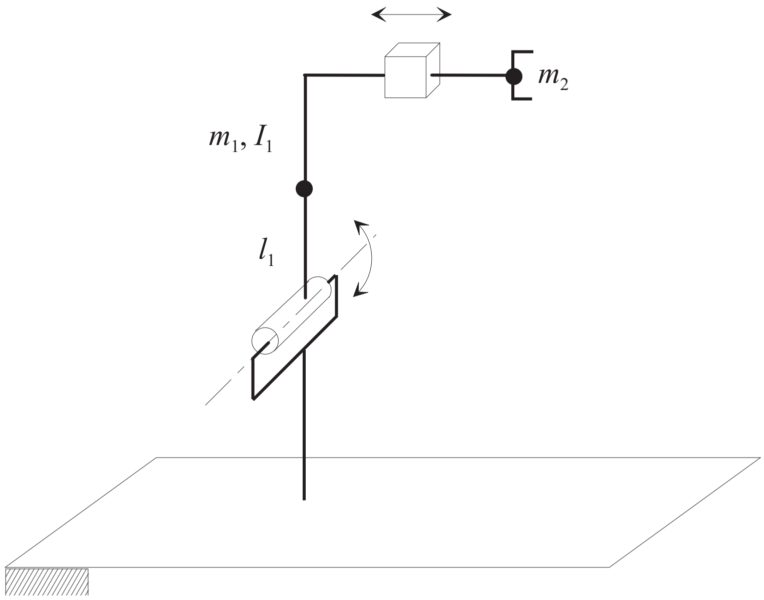
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[2] for this manipulator.

3. Ignoring the gravitational terms, compute the dynamic model of this manipulator.

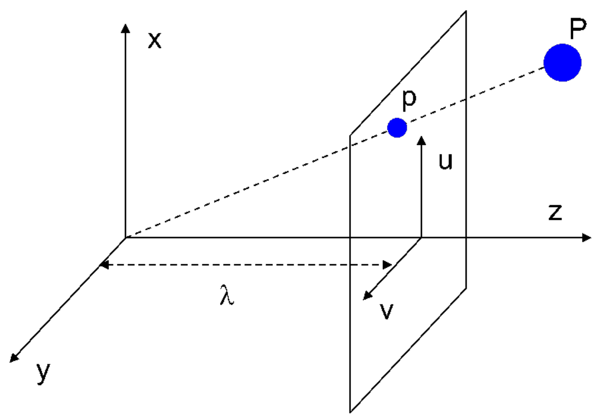
[2]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Write the dynamic model in a linear form with respect to a set of dynamic parameters.

EXERCISE 2

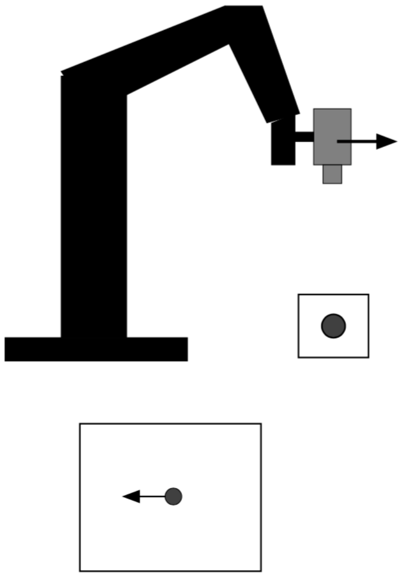
Consider the control of a manipulator with vision sensors.

1. Explain what is the "perspective projection" method and, making reference to the following picture, write the related formulas.



2. Making reference to the following picture, where a single image point is considered, explain what is the interaction matrix in the context of visual control, specifying precisely:

    - the variables that are related by the interaction matrix
    - the size of the interaction matrix
    - the variables upon which the interaction matrix depends



3. Suppose now that there are n image points: explain how to build the interaction matrix and what is the size of such matirx in this case.

4. Explain what is the image Jacobian and what is its relation with the interaction matrix.

EXERCISE 3

1. Given a kinematic constraint in Pfaffian form

$$X(\mathbf{q})\,\dot{x} + Y(\mathbf{q})\,\dot{y} + Z(\mathbf{q})\,\dot{z} = 0$$

   where $\mathbf{q} = \begin{bmatrix} x & y & z \end{bmatrix}$ is the configuration vector. Illustrate the necessary and sufficient condition for this constraint to be holonomic.

2. Consider the following kinematic constraints

$$\dot{q}_1 + q_1\dot{q}_2 - \dot{q}_3 = 0 \qquad q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$$

   where $\mathbf{q} \in \mathbb{R}^3$ is the configuration vector. Determine, using the necessary and sufficient condition, if each of these constraints by itself is holonomic or nonholonomic.

3. Consider two mobile robots, whose configurations are described by $\mathbf{q} \in \mathbb{R}^3$, and whose motion is described by $\dot{q}_1 + q_1\dot{q}_2 - \dot{q}_3 = 0$ for the first robot, and $q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$ for the second robot.
Does the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} -q_1 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u_2$$

describe the motion of the first robot, and the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} q_2^2 \\ 0 \\ q_1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

the motion of the second robot?
Clearly motivate the answer.

4. Considering the second constraint ($a^T(\mathbf{q})\dot{\mathbf{q}} = q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$), two vectors in the null of $a^T(\mathbf{q})$ are

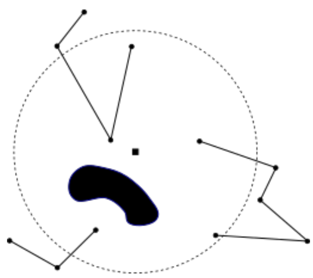
$$\mathbf{g}_1 = \begin{bmatrix} \frac{q_2^2}{q_1} \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{g}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Compute the vector field representing the motion that is locally constrained by $q_1\dot{q}_1 - q_2^2\dot{q}_3 = 0$.

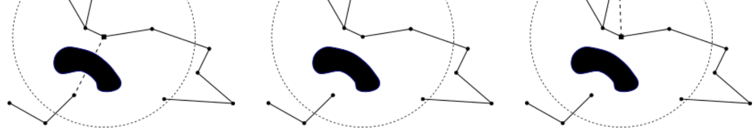
EXERCISE 4

1. Write and explain the pseudocode of the algorithm to construct the probabilistic roadmap used by PRM.

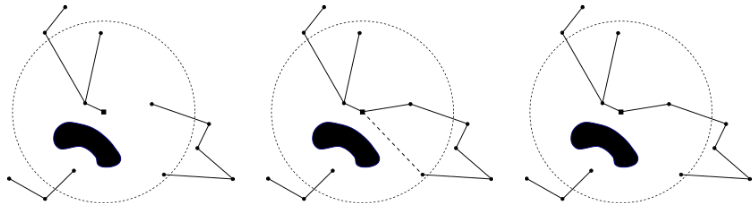
2. Consider the i-th iteration of PRM as depicted in the figure below



where the black square is the current $\mathbf{q}_{rand}$, the black blob region is an obstacle, the black dashed circle centred in $\mathbf{q}_{rand}$ is the region defining the Near set, and black dots and segments are nodes and edges in the actual V and E sets, respectively.
Put the following pictures in the correct order, according to the execution of the algorithm across the nodes belonging to the Near set (black dashed edges represent connection attempts that are discarded).



A B C



D E F

3. How could the path resulting from PRM be used to speed up the computation of RRT*? Clearly motivate the answer.

4. Consider again an RRT* planner, what are the characteristics a cost function should satisfy in order to be included into the planner?