Control of Industrial and Mobile Robots

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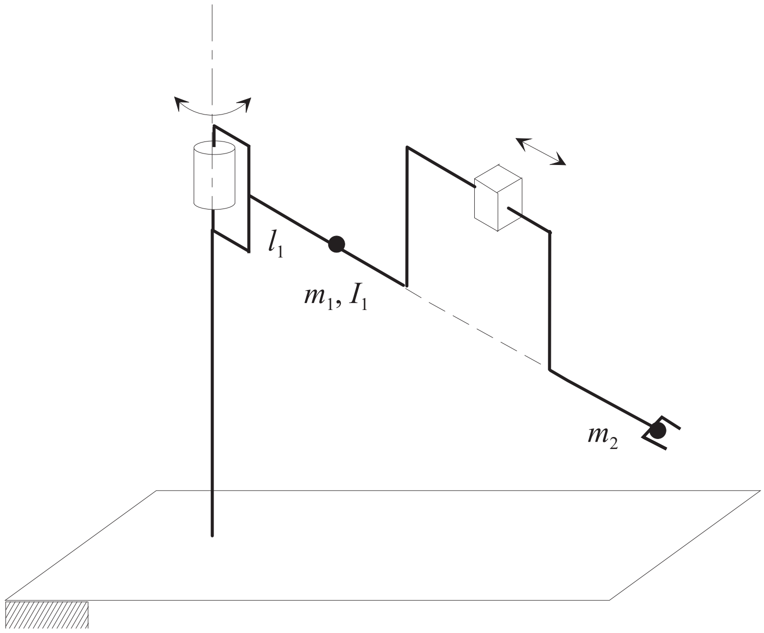
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 90 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Write the dynamic model for this manipulator.

[1]The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For a generic manipulator without gravitational and friction effects, show that the equation:

$$\dot{\mathbf{q}}^T \left(\dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \right) \dot{\mathbf{q}} = 0$$

is valid for any choice of the Coriolis and centrifugal matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$.

EXERCISE 2

Consider a kinematically redundant manipulator.

1. Write the general expression of the solutions of the inverse kinematics problem at velocity level.

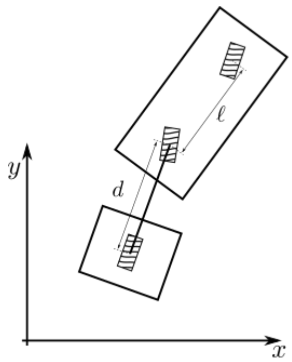
2. Consider the weighted pseudo inverse method for the solution of the inverse kinematics of a redundant manipulator. Write the expression of the cost function and discuss a criterion to select the weights.

3. The inverse kinematics at velocity level for a redundant manipulator is often implemented in closed-loop. Explain the reason for this and sketch the block diagram for such a closed-loop scheme.

4. Consider now motion planning of the end-effector position. If the end-effector task is expressed in terms of position only, what is the minimum number of joints for the manipulator to be redundant with respect to this task?

EXERCISE 3

Consider a car-trailer system, constituted by a trailer equipped with a fixed wheel and a car equipped with two steering wheels, shown in the figure below.



1. Determine the configuration vector and show the configuration variables on the figure above.

2. Derive the kinematic constraints that allow to determine the kinematic model of the car-trailer system, and write them in Pfaffian form.

3. Consider the following equations

$$
\begin{aligned}
\dot{x} &= v\, d\ell \cos(\phi_1) \cos(\theta_t + \phi_2) \\
\dot{y} &= v\, d\ell \cos(\phi_1) \sin(\theta_t + \phi_2) \\
\dot{\theta} &= v\, d \sin(\phi_1 - \phi_2) \\
\dot{\theta}_t &= v\, \ell \cos(\phi_1) \sin(\theta_t - \theta - \phi_2) \\
\dot{\phi}_1 &= \omega_1 \\
\dot{\phi}_2 &= \omega_2
\end{aligned}
$$

Is this the kinematic model of the car-trailer system? Clearly motivate the answer and support it with a theoretical proof.

4. Consider now the car without the trailer, and assume the rear wheel is fixed while the front is steerable. The car velocity and front steering position are constrained as follows

$$
0 \le v \le \bar{v} \qquad \bar{\phi}_m \le \phi \le \bar{\phi}_M
$$

How should we limit the linear and angular velocity of the canonical model, in order to be consistent with these constraints?

EXERCISE 4

1. Two important parts of an autonomous navigation system are the local and the global planner. What are the most important characteristics of these two functionalities?

2. Explain what are the advantages of structuring the navigation system in a hierarchical way, separating local from global planning.

3. Consider now as global planner RRT*. Write the pseudocode of the rewire procedure, and explain the role of this procedure inside the RRT* algorithm and its connection with optimality.

4. RRT* is used to plan the path of an electric vehicle, whose battery pack can be recharged when the vehicle travels a hill. Can the function

$$c(\sigma) = \text{length}(\sigma) + \text{discharge}(\sigma)$$

where σ is a path, lenth is the function that computes the length of a path, and discharge the function that computes the battery discharge (or charge if the value is negative) that occurs travelling a path, be used as cost for the planner?