Control of Industrial and Mobile Robots

Prof. Rocco, Bascetta

July 6, 2026

NAME:
UNIVERSITY ID NUMBER:

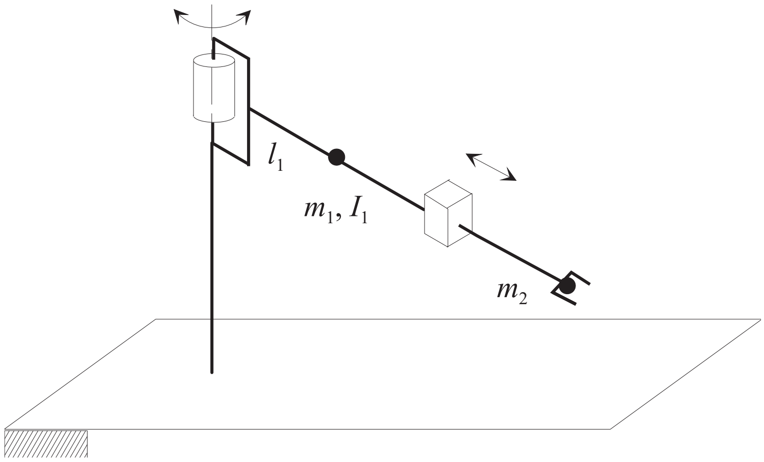
SIGNATURE: ______________________________

Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator. Is this the only possible expression that matrix $\mathbf{C}$ can take?

3. Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew symmetric.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. Consider now the expression $\dot{\mathbf{q}}^T \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} = 0$. Explain whether this result can be obtained only with a matrix $\mathbf{C}$ computed from the Christoffel symbols or if it is general.

EXERCISE 2

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta \mathbf{x} = \mathbf{\Phi} \Delta \zeta$$

   Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

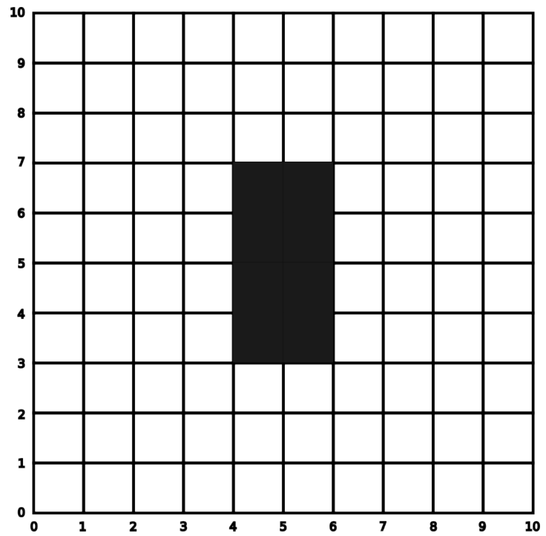
3. In the kinematic calibration, the pseudoinverse of a matrix is used. Explain precisely what matrix is this and whether the right or left pseudoinverse is used.

4. In order to find the solution of the inverse kinematics for a redundant robot, a pseudoinverse matrix is used as well. Explain what matrix is this and whether the right or left pseudoinverse is used in this case.

EXERCISE 3

1. Write the pseudocode for the roadmap construction of sPRM planning algorithm.

2. Consider the 2D environment depicted in the picture below, where the black areas represent obstacles. Randomly drawing from the configuration space gives rise to the following sequence $\mathbf{q}_1 = [2, 2]$, $\mathbf{q}_2 = [2, 8]$, $\mathbf{q}_3 = [5, 1]$, $\mathbf{q}_4 = [8, 2]$, $\mathbf{q}_5 = [8, 8]$, $\mathbf{q}_6 = [5, 9]$, $\mathbf{q}_7 = [9, 5]$, $\mathbf{q}_8 = [9, 9]$.
   Using sPRM, with a radius of 4.5, determine a roadmap, and draw it in the picture below.



3. Considering the roadmap constructed in the previous question, determine the path starting from $\mathbf{q}_{\text{start}} = [0, 0]$ and reaching $\mathbf{q}_{\text{goal}} = [10, 10]$ composed by the minimum number of nodes. Write the ordered sequence of nodes representing the path.

4. Considering the same $\mathbf{q}_{\text{start}}$ and $\mathbf{q}_{\text{goal}}$, are the following paths

$$\mathbf{q}_{\text{start}} - \mathbf{q}_1 - \mathbf{q}_3 - \mathbf{q}_4 - \mathbf{q}_7 - \mathbf{q}_5 - \mathbf{q}_8 - \mathbf{q}_{\text{goal}}$$
$$\mathbf{q}_{\text{start}} - \mathbf{q}_1 - \mathbf{q}_3 - \mathbf{q}_4 - \mathbf{q}_7 - \mathbf{q}_5 - \mathbf{q}_6 - \mathbf{q}_8 - \mathbf{q}_{\text{goal}}$$
$$\mathbf{q}_{\text{start}} - \mathbf{q}_1 - \mathbf{q}_3 - \mathbf{q}_4 - \mathbf{q}_7 - \mathbf{q}_5 - \mathbf{q}_2 - \mathbf{q}_6 - \mathbf{q}_8 - \mathbf{q}_{\text{goal}}$$

solutions of the planning problem determined by sPRM?

EXERCISE 4

1. Are the following sentences true or false?

		T	F
(A)	A forward-only car is STLA but it is not STLC, a car with reverse gear is STLC	☐	☐
(B)	When exact linearization is applied to the unicycle or bicycle model, the orientation of the robot can no longer be controlled	☐	☐
(C)	The canonical simplified model can be transformed into two double integrators representing the motion of a point P that can be selected at any location on the chassis of the robot	☐	☐
(D)	Odometric localization is subject to a drift that increases as the length of the path increases	☐	☐

2. Consider a canonical simplified model linearized using two different controllers, one derived selecting point P along the direction of the forward velocity vector, and another one selecting point P in an arbitrary point of the chassis of the robot. Simulate the motion of the robot, starting from the origin with orientation 225°, and selecting $v_{xP}(t) = v_{yP}(t) = 2.5\,\mathrm{m\,s^{-1}}$. Is the motion performed by the robot the same for the two controllers?

3. Consider a canonical simplified model linearized selecting a point P along the direction of the forward velocity vector. The robot starts from the origin with orientation $60°$, and is fed by $v_{xP}(t) = 0.5\,\mathrm{m\,s^{-1}}$ and $v_{yP}(t) = 0.5\sqrt{3}\,\mathrm{m\,s^{-1}}$.
Write the values of the position and orientation of the robot at $t_1 = 1\,\mathrm{s}$, $t_2 = 5\,\mathrm{s}$, $t_3 = 10\,\mathrm{s}$.

4. Consider a bicycle robot described by the following kinematic model

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta \qquad \dot{\theta} = v\tan\varphi/\ell$$

Given the wheel positions and the steering angle measurement, write the equations that allow to estimate the pose of the robot using the second-order Runge-Kutta integration method.