Control of Industrial and Mobile Robots

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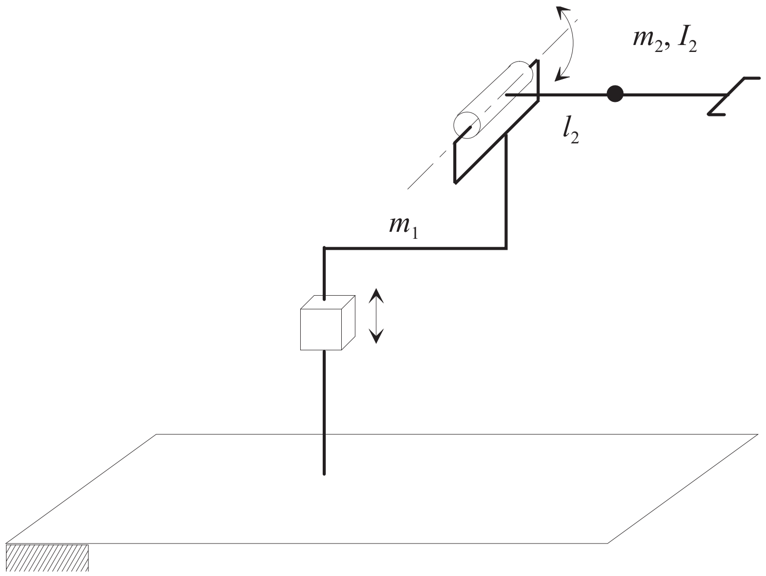
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

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[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

2. Compute the gravitational terms for this robot.

3. Ignoring the Coriolis and centrifugal terms, write the dynamic model of the manipulator and show that this model is linear with respect to a certain set of dynamic parameters.

4. The linearity of the model in a set of dynamic parameters allows to setup experiments for the identification of such parameters. For a generic manipulator, explain what are the variables that need to be recorded during the experiments. With reference to the dynamic model of this exercise, is it possible to experimentally identify the mass of the first link?

EXERCISE 2

1. Explain what is the purpose of the kinematic calibration of a robot manipulator and why it is needed.

2. In the kinematic calibration of a robot manipulator the following equation is used:

$$\Delta \mathbf{x} = \mathbf{\Phi} \Delta \zeta$$

Explain the meaning of each symbol used in such equation, as well as the size of the vectors.

3. Based on the above equation, explain how the kinematic calibration can be performed.

4. Consider now a kinematically redundant manipulator. Write the general solution of the inverse kinematics at velocity level. Is the pseudoinverse matrix that appears in this equation the same pseudoinverse of the kinematic calibration problem?

EXERCISE 3

Consider the following system of kinematic constraints

$$
\begin{aligned}
2\dot{q}_1 + q_1\dot{q}_2 - 3\dot{q}_3 &= 0 \\
2\dot{q}_2 - q_2\dot{q}_3 &= 0
\end{aligned}
$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T$.

1. Using the necessary and sufficient condition, determine if the first constraint, considered as an independent constraint, is holonomic or nonholonomic.

2. Is the second constraint, considered as an independent constraint, holonomic or nonholonomic?

3. Assuming that $A^T(\mathbf{q})\,\dot{\mathbf{q}} = 0$ is the Pfaffian form of the system of two constraints, and

$$g_1(\mathbf{q}) = \begin{bmatrix} 6 - q_1 q_2 \\ 2q_2 \\ 4 \\ 0 \end{bmatrix} \qquad g_2(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

are two vectors in the null space of $A^T(\mathbf{q})$, demonstrate that the system of two constraints is holonomic.

4. Does the following kinematic model

$$\dot{\mathbf{q}} = g_1(\mathbf{q})\,u_1 + g_2(\mathbf{q})\,u_2$$

where $g_1(\mathbf{q})$ and $g_2(\mathbf{q})$ are the vectors introduced in the previous step, describe the motion of the mobile robot characterized by the system of two constraints?

EXERCISE 4

Consider the design of the trajectory tracking controller of a robot described by the rear-wheel-drive bicycle model.

1. Write the relations that allow to transform the bicycle model into the canonical simplified model. Under which assumptions do these relations hold?

2. Considering a point P, located along the linear velocity vector v at a distance ε from the wheel contact point, write the analytical expression of a feedback linearizing controller for the canonical simplified model.

3. Using the transformation derived in step 1, write the analytical expression of the feedback linearizing controller for the bicycle model using the relations of the feedback linearizing controller for the canonical simplified model. Write the detailed procedure, not only the results.

4. Consider an implementation of the controller as a ROS node. Assuming it receives the robot actual pose measurement as a *Float64MultiArray* message, where the array elements are x, y, θ, and publishes the robot commands as a *Float64MultiArray* message, where the array elements are v and ϕ, complete the code of the callback in order to store the actual pose in the class variables *act_pose_x*, *act_pose_y*, *act_pose_theta*. Write the code to compute the reference velocities, v_{x_P} and v_{y_P}, as unitary steps starting after 5 seconds, and the vehicle commands, using function

```
void canonical_controller::bicycle_feedback_linearization(double vPx,
    double vPy, double& v, double& phi)
```

that is already implemented, and to publish them using an already defined publisher *vehicleCommand_publisher*.

```
void canonical_controller::vehiclePose_MessageCallback(const std_msgs::
    Float64MultiArray::ConstPtr& msg)
{







}

void canonical_controller::PeriodicTask(void)
{

}
```