Control of Industrial and Mobile Robots

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UNIVERSITY ID NUMBER:

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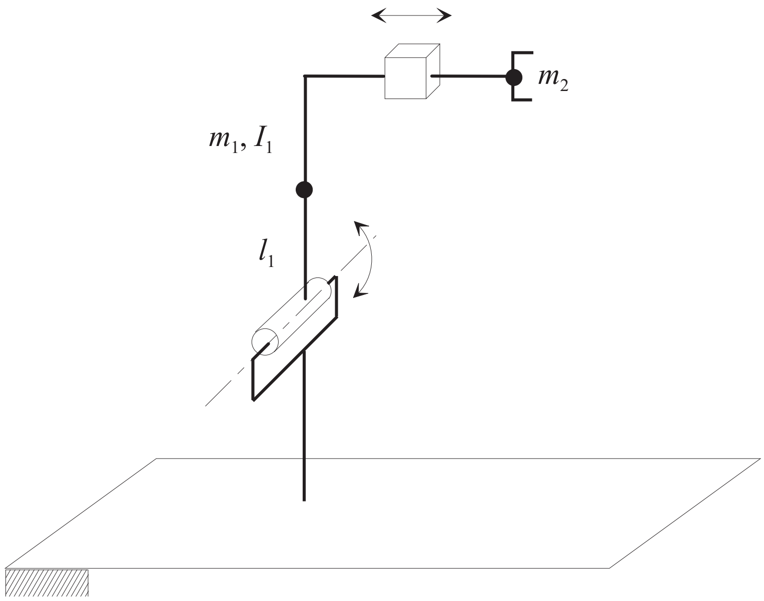
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Compute the matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ of the Coriolis and centrifugal terms[1] for this manipulator.

3. Ignoring the gravitational terms, write the complete dynamic model for this manipulator.

[1] The general expression of the Christoffel symbols is $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

4. For this specific manipulator, wtite the expression of the kinetic energy. Is it possible that this kinetic energy is zero for joint velocities different from zero?

EXERCISE 2

1. Explain why for the kinematic scaling of trajectories it is possible to consider each joint separately, while for the dynamic scaling this is not possible.

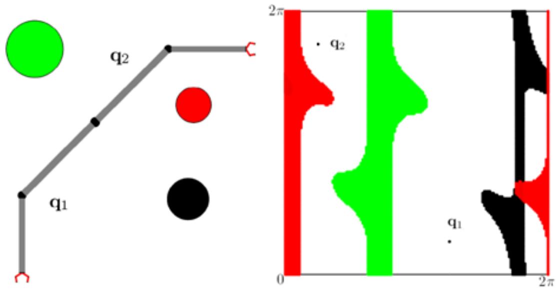
2. Consider the following equation:

$$\tau_i(t) = \alpha_i\left(r(t)\right)\ddot{r}(t) + \beta_i\left(r(t)\right)\dot{r}^2(t) + \gamma_i\left(r(t)\right)$$

   Explain whether such equation is used in the kinematic scaling or in the dynamic scaling and define all symbols used in the equation.

3. Assume now that, in a robot that is not affected by gravity, trajectories have been planned such that the torque of one joint exceeds its limit by 44% (the torques of the other joints are within their limits). Explain how the trajectory can be scaled and what should be the scaling factor. What is meant with "variable scaling" in this context?

4. With specific reference to the following picture, define the concept of "configuration space" in the path planning with obstacle avoidance problem. What do the irregular shapes in the right hand side picture represent?



EXERCISE 3

1. Consider the following kinematic constraint

$$2x^2\dot{y} - 4(x-1)\dot{z} + 3\dot{w} = 0$$

where $\mathbf{q} = \begin{bmatrix} x & y & z & w \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

2. Consider the following kinematic constraint

$$3\dot{x} - (x-1)\dot{y} + 2\dot{z} = 0$$

where $\mathbf{q} = \begin{bmatrix} x & y & z & w \end{bmatrix}$ is the configuration vector. Determine, using the necessary and sufficient condition, if this constraint is holonomic or nonholonomic.

3. Consider the system of kinematic constraints in Pfaffian form $A^T(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}$, where $A^T(\mathbf{q})$ is a 2×4 matrix and $\operatorname{rank}\left(A^T(\mathbf{q})\right) = 2$. Answer the following questions:

   (a) what is the dimension of the configuration space?

   (b) how many constraints are we considering?

   (c) what is the dimension of the null space?

   (d) what values the dimension of the accessibility distribution can take?

   (e) for which of these values the set of constraints is nonholonomic and for which holonomic?

4. Consider a mobile robot whose configuration is represented by $\mathbf{q} \in \mathbb{R}^4$, and whose motion is described by the system of kinematic constraints of questions 1 and 2. Show that the following kinematic model

$$\dot{\mathbf{q}} = \begin{bmatrix} 2(2x-1) \\ 6(1-x) \\ -3x^2 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} x-1 \\ 3 \\ 0 \\ -2x^2 \end{bmatrix} u_2$$

describes the motion of the robot.

EXERCISE 4

1. The motion of a robot can be represented using the canonical simplified model. Considering a point P at a distance ε from the wheel contact point, along the direction of the robot velocity vector, write the equations of the feedback linearizing controller that allows to linearize the canonical simplified model.

2. Write the analytical equations of the complete trajectory tracking controller, that allows to compute v and ω given the desired trajectory $x^d(t)$, $y^d(t)$, $\theta^d(t)$, and a measure of the robot position and heading, $x(t)$, $y(t)$, $\theta(t)$. The position controller is a PI controller.

3. Consider an implementation of this controller as a ROS node. Assuming the controller receives the robot actual pose measurement as a *Float64MultiArray* message, where the array elements are x, y, θ, and publishes the robot commands as a *Float64MultiArray* message, where the array elements are v and ω, complete the code of the callback in order to store the actual pose in the node variables

act_pose_x, *act_pose_y*, *act_pose_theta*, and write the code to fill in the vehicle commands message and to publish it using an already defined publisher *vehicleCommand_publisher*, and the values stored in the node variables *act_v* and *act_omega*.

```
void canonical_controller::vehiclePose_MessageCallback(const std_msgs::
    Float64MultiArray::ConstPtr& msg)
{
    // Input data: x, y, theta







}

void canonical_controller::PeriodicTask(void)
{
    // ...

    /*  Publish vehicle commands */







}
```

4. Assuming now the motion of the robot can be represented using the bicycle kinematic model, with a steering angle that can be changed instantaneously. Write the relations that allow to apply the controller developed in question 2 to the bicycle robot.