# Control of Industrial and Mobile Robots

Prof. Rocco, Bascetta

July 23, 2024

NAME:
UNIVERSITY ID NUMBER:

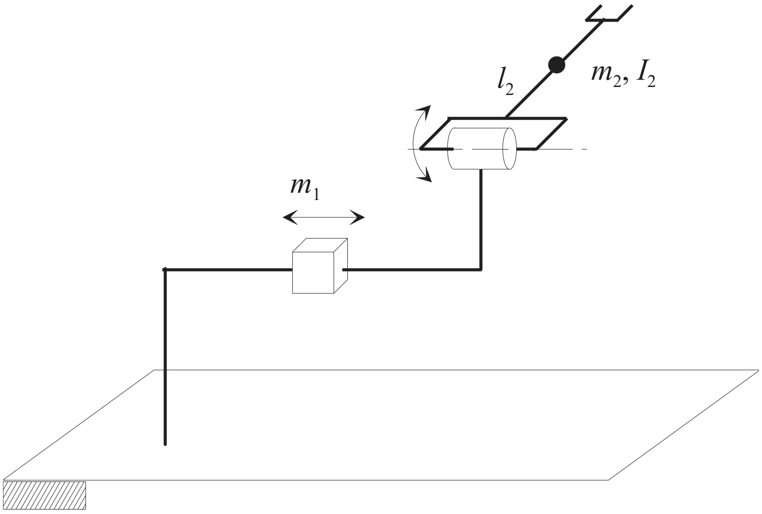
SIGNATURE: ______________________________

**Warnings**

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

## EXERCISE 1

1. Consider the manipulator sketched in the picture:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator.

2. Write the complete dynamic model for this manipulator.

3. Consider the adoption of an inverse dynamics controller for this manipulator. Write the expressions of the two control variables.

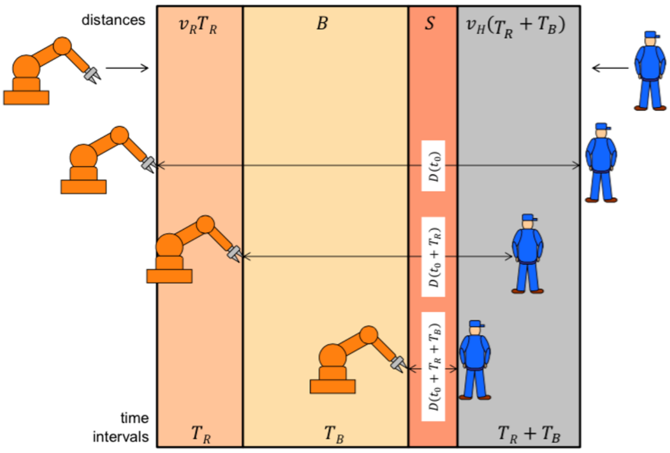
4. Assume that the inverse dynamics controller assigns the same dynamics in closed loop to both joints of the manipulator. Compute the gains of the controller in such a way that one eigenvalue is in $-10$ and the other one is in $-20$.

**EXERCISE 2**

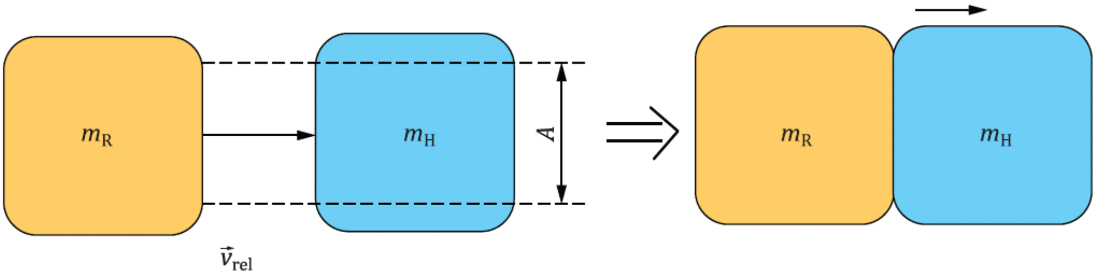
1. With reference to collaborative robotics, explain what are the general advantages in the use of cobots for small and medium enterprises.

2. One of the modes of interaction between human and robot allowed by the safety standards is the hand guiding. Explain what is the control system methodology that allows to hand guide an industrial robot. What variables need to be measured in order to implement such method?

3. Making reference to the following picture, write the inequality that has to be satisfied according to the speed and separation monitoring safety standard, explaining the meaning of the symbols used. What is a standard value assumed for the human velocity, in case it is not monitored?

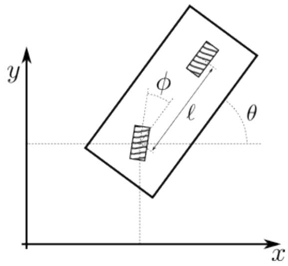


4. Consider now a robot that is compliant with the power and force limiting safety standards. Making reference to the following picture, derive the expression of the maximum value of the relative velocity between robot and human requested by such standard.



## EXERCISE 3

Consider a front-wheel drive bicycle with rear steerable wheel, as shown in the picture below.



1. Write the expression of vector $\mathbf{q}$ representing the configuration of the robot, and select the inputs, in accordance with the standard bicycle, that should be considered in order to write the kinematic model.

2. Derive the kinematic constraints that describe the motion of the bicycle and write them in Pfaffian form.

3. Write the first two equations of the kinematic model of the front-wheel drive bicycle, describing the evolution in the $xy$ plane of the position of its rear wheel contact point (i.e., $\dot{x} = \dots$, $\dot{y} = \dots$).

4. Does the following equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \dfrac{\ell \cos\theta}{\tan\phi} \\ \dfrac{\ell \sin\theta}{\tan\phi} \\ 1 \\ 0 \end{bmatrix} \alpha + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega$$

represent the kinematic model of the front-wheel drive bicycle with rear steerable wheel?

If yes, what physical quantity does $\alpha$ represent? and how can the relation $v = \dfrac{\ell}{\tan\phi}\alpha$ be explained using the ICR?

## EXERCISE 4

1. Consider a unicycle mobile robot. Selecting as flat outputs $z_1 = x$ and $z_2 = y$, write the flat model of the robot, i.e., the analytical relations from $z_1$, $z_2$ to $x$, $y$, $\theta$ and from $z_1$, $z_2$ to $v$, $\omega$.

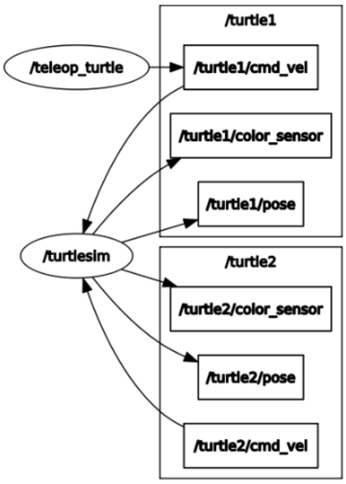
2. Using the flatness transformation, determine the analytic expression of a trajectory $x(t)$, $y(t)$ (and the numerical values of its coefficients) that moves a unicycle robot, in an obstacle free environment, from an initial state $x_i = y_i = \theta_i = 0$ and $v_i = 0$ at $t_i = 0$, to a final state $x_f = y_f = 2$, $\theta_f = 0$ and $v_f = 0$ at $t_f = 1$.

3. Modify the answer to the previous step in order to introduce the minimization of the cost

$$J(v, \omega) = \int_0^{T_f} \left(v^2 + 0.5\omega^2\right) \, \mathrm{d}t$$

where now $T_f$ is a free parameter. Write the analytical expression of the relations that allow to compute the additional coefficients that must be introduced in order to enforce the minimization of the cost function.

4. Consider a ROS robot simulator, whose architecture is represented by the following graph.



List the nodes and topics in the graph. Which topics allow nodes to communicate? Using which messages (specify the message type)?