Control of Industrial and Mobile Robots

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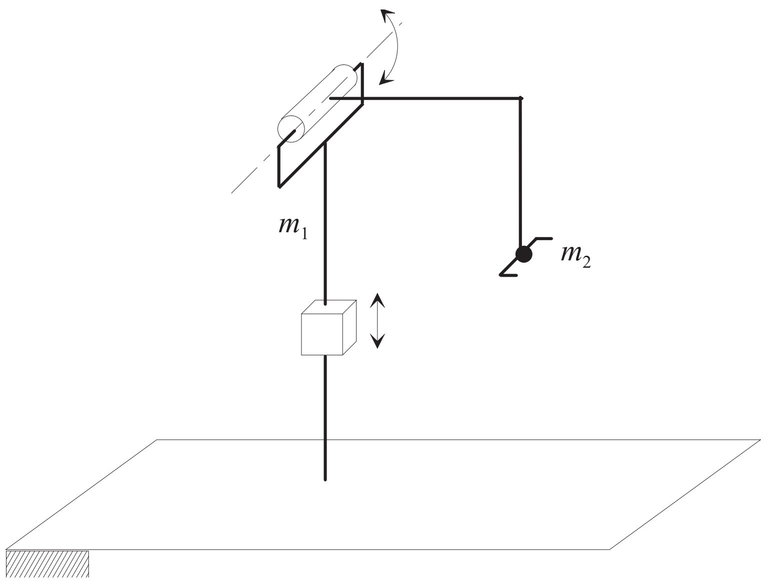
Warnings

- This file consists of **10** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix $\mathbf{B}(\mathbf{q})$ of the manipulator[1].

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[1]The cross product between vector $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix}$

2. Compute the vector of the gravitational terms for this manipulator.

3. Write the expression of the PD + gravity compensation control law for this manipulator.

4. What is the theoretical property guaranteed by a PD + gravity compensation control scheme?

EXERCISE 2

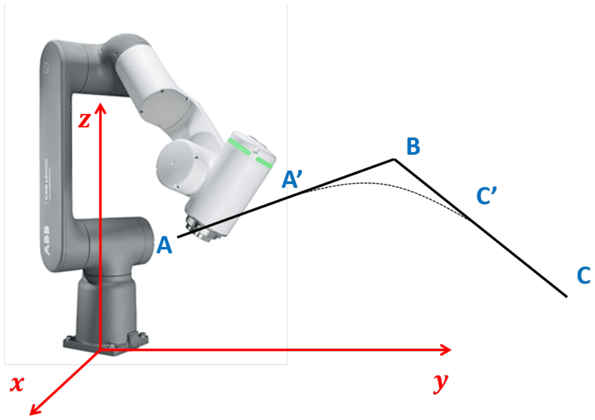
1. Suppose that a trajectory for a scalar variable has to be defined, which achieves the values reported in the following table, at the given instants:

$$\begin{array}{lllll} t_1 = 0 & t_2 = 3 & t_3 = 5 & t_4 = 7 & t_5 = 10 \\ q_1 = 30 & q_2 = 0 & q_3 = 20 & q_4 = 50 & q_5 = 65 \end{array}$$

   Assume that you want to use cubic polynomials in each interval. Assign suitable values to the speed at the intermediate points.

2. Consider now the interpolation of such points by a single polynomial of suitable degree. What are the issues that suggest not to use this solution?

3. The concatenation of linear paths is a common practice in robotic motion planning. Making reference to the following picture, and without going through all the mathematics, explain what are the considerations that can be done in order to design such concatenation.



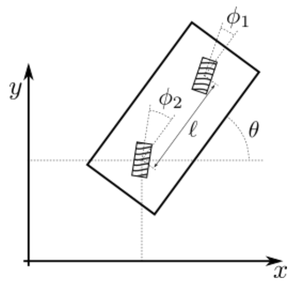
4. Consider the following instructions in the ABB robot programming language:

```
MoveL p10, v200, fine;
MoveL p20, v200, z20;
MoveL p30, v200, fine;
MoveL p40, v200, z50;
MoveL p10, v200, fine;
```

Briefly explain what is the meaning of such instructions.

EXERCISE 3

Consider a rear-wheel drive bicycle with front and rear steerable wheels, as shown in the picture below.



1. Write the expression of vector $\mathbf{q}$ representing the configuration of the robot, and select the inputs, in accordance with the standard bicycle, that should be considered in order to write the kinematic model.

2. Derive the kinematic constraints that describe the motion of the bicycle and write them in Pfaffian form.

3. Write the first two equations of the kinematic model of the rear-wheel drive bicycle, describing the evolution in the xy plane of the position of its rear wheel contact point (i.e., $\dot{x} = \ldots$, $\dot{y} = \ldots$).

4. Does the following equation

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\ell \cos \phi_1 \cos (\theta + \phi_2)}{\sin (\phi_1 - \phi_2)} \\ \dfrac{\ell \cos \phi_1 \sin (\theta + \phi_2)}{\sin (\phi_1 - \phi_2)} \\ 1 \\ 0 \\ 0 \end{bmatrix} \alpha + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \omega_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega_2$$

represent the kinematic model of the rear-wheel drive bicycle with front and rear steerable wheels? If yes, what physical quantity does α represent? and how can the relation $v = \dfrac{\ell \cos \phi_1}{\sin (\phi_1 - \phi_2)} \alpha$ be explained?

EXERCISE 4

Consider the design of a trajectory tracking controller for a unicycle robot based on feedback linearization.

1. Given a point P located along the linear velocity vector, at a distance ε from the wheel contact point, derive the control law of a feedback linearizing controller able to guarantee that the robot tracks the trajectory with a forward motion.

2. Using the control law derived in the previous step and the unicycle kinematic model, derive the expression of the dynamic system representing the closed-loop system obtained connecting the linearizing controller and the kinematic model (including any hidden state).

3. Write the analytical expression of a simple linear trajectory tracking controller, based on a proportional controller with velocity feed-forward. Draw the block diagram including the robot model, the feedback linearizing controller, and the trajectory tracking controller.

4. Assume now that the robot pose used by the trajectory tracking controller is estimated using a simple odometric localization algorithm. Write the equations that, at each time step (t_k), allow to estimate the robot pose (x_k, y_k, θ_k) starting from the measured wheel rotations $\Delta\phi_{R_k}$ and $\Delta\phi_{L_k}$.