Control of industrial robots

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Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
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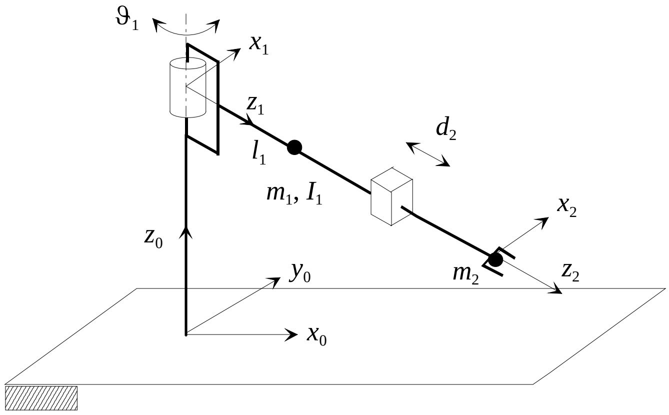
Exercise 1

Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector



1.1 Find the expression of the inertia matrix of the manipulator.

Denavit-Hartenberg frames can be defined as sketched in the picture.

Computations of the Jacobians:

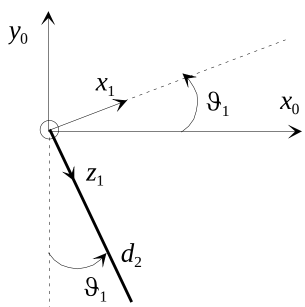
Link 1

$$J_P^{(l_1)} = \begin{bmatrix} j_{P1}^{(l_1)} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} z_0 \times (p_{l1} - p_0) & \mathbf{0} \end{bmatrix} = \begin{bmatrix} l_1 c_1 & 0 \\ l_1 s_1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_O^{(l_1)} = \begin{bmatrix} j_{O1}^{(l_1)} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} z_0 & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Link 2

$$J_P^{(l_2)} = \begin{bmatrix} j_{P1}^{(l_2)} & j_{P2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} z_0 \times (p_{l2} - p_0) & z_1 \end{bmatrix} = \begin{bmatrix} d_2 c_1 & s_1 \\ d_2 s_1 & -c_1 \\ 0 & 0 \end{bmatrix}$$



Auxiliary vectors for the above computations:

$$p_{l1} = \begin{bmatrix} l_1 s_1 \\ -l_1 c_1 \\ * \end{bmatrix}, \quad p_{l2} = \begin{bmatrix} d_2 s_1 \\ -d_2 c_1 \\ * \end{bmatrix}, \quad z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

Inertia matrix:

$$B(q) = m_1 J_P^{(l_1)T} J_P^{(l_1)} + I_1 J_O^{(l_1)T} J_O^{(l_1)} + m_2 J_P^{(l_2)T} J_P^{(l_2)} = m_1 \begin{bmatrix} l_1^2 & 0 \\ 0 & 0 \end{bmatrix} + I_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} d_2^2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

$$b_{11} = m_1 l_1^2 + I_1 + m_2 d_2^2$$

$$b_{12} = 0$$

$$b_{22} = m_2$$

1.2 Compute the matrix $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ of the Coriolis and centrifugal terms [1] for this manipulator

The only derivative in the Christoffel symbols which is different from zero is:

$$\frac{\partial b_{11}}{\partial q_2} = 2m_2 d_2$$

Therefore:

$$c_{111} = 0 \qquad c_{211} = -\frac{1}{2}\frac{\partial b_{11}}{\partial q_2} = m_2 d_2$$

$$c_{112} = c_{121} = \frac{1}{2}\frac{\partial b_{11}}{\partial q_2} = m_2 d_2 \qquad c_{212} = c_{221} = 0$$

$$c_{122} = 0 \qquad c_{222} = 0$$

The matrix of the Coriolis and centrifugal terms is thus:

$$\boldsymbol{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \qquad \begin{aligned} c_{11} &= c_{111}\dot{q}_1 + c_{112}\dot{q}_2 = m_2 d_2 \dot{d}_2 \\ c_{12} &= c_{121}\dot{q}_1 + c_{122}\dot{q}_2 = m_2 d_2 \dot{\vartheta}_1 \\ c_{21} &= c_{211}\dot{q}_1 + c_{212}\dot{q}_2 = -m_2 d_2 \dot{\vartheta}_1 \\ c_{22} &= c_{221}\dot{q}_1 + c_{222}\dot{q}_2 = 0 \end{aligned}$$

1.3 Write the complete dynamic model for this manipulator

Since the motion is in a horizontal plane, the gravitational terms are clearly zero.

Thus the model can be written as:

$$\boldsymbol{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \boldsymbol{\tau}$$

or

$$\left(m_1 l_1^2 + I_1 + m_2 d_2^2\right)\ddot{\vartheta}_1 + 2m_2 d_2 \dot{\vartheta}_1 \dot{d}_2 = \tau_1$$

$$m_2 \ddot{d}_2 - m_2 d_2 \dot{\vartheta}_1^{\,2} = \tau_2$$

1.4 Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters.

It is possible to express the dynamic model as:

$$\boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\pi} = \boldsymbol{\tau}$$

where

$$\boldsymbol{\pi} = \begin{bmatrix} m_1 l_1^2 + I_1 \\ m_2 \end{bmatrix}, \quad \boldsymbol{Y} = \begin{bmatrix} \ddot{\vartheta}_1 & d_2^2\ddot{\vartheta}_1 + 2d_2\dot{\vartheta}_1\dot{d}_2 \\ 0 & \ddot{d}_2 - d_2\dot{\vartheta}_1^{\,2} \end{bmatrix}$$

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[1] The general expression of the Christoffel symbols is: $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

Exercise 2

Consider a kinematically redundant manipulator

2.1 Write the general expression of the solutions of the inverse kinematics problem at velocity level

Given a set of desired task variables $\boldsymbol{r}_d$, such that:

$$\dot{\boldsymbol{r}}_d = \boldsymbol{J}\dot{\boldsymbol{q}}$$

where $\boldsymbol{J}$ is the Jacobian matrix, the general expression of the solution of the inverse kinematics is:

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}\dot{\boldsymbol{r}}_d + \boldsymbol{P}\dot{\boldsymbol{q}}_0$$

$\boldsymbol{J}^{\#}$ is the pseudo-inverse of the Jacobian, defined as:

$$\boldsymbol{J}^{\#} = \boldsymbol{J}^T\left(\boldsymbol{J}\boldsymbol{J}^T\right)^{-1}$$

The term $\boldsymbol{P}\dot{\boldsymbol{q}}_0$ defines the null-space motions: $\boldsymbol{P}$ is a matrix that projects a generic joint velocity $\dot{\boldsymbol{q}}_0$ in the null space of the Jacobian and takes the expression:

$$\boldsymbol{P} = \boldsymbol{I}_n - \boldsymbol{J}^{\#}\boldsymbol{J}$$

2.2 Express the solution in the form that includes a closed loop correction (kinematic control) and explain why this correction is used

The solution with closed-loop correction can be written as:

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}(\boldsymbol{q})\left[\dot{\boldsymbol{r}}_d + \boldsymbol{K}(\boldsymbol{r}_d - \boldsymbol{r})\right] + \left(\boldsymbol{I}_n - \boldsymbol{J}^{\#}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})\right)\dot{\boldsymbol{q}}_0$$

where:

$$\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{q})$$

is obtained through direct kinematics.

The correction is used to recover errors with respect to an assigned task $\boldsymbol{r}_d$ due to initial mismatches, drifts, inaccuracies of the solution.

2.3 Explain some criteria to select the joint velocities to be projected onto the null space of the Jacobian matrix.

One possible choice for the velocities $\dot{\boldsymbol{q}}_0$ is the projected gradient method:

$$\dot{\boldsymbol{q}}_0 = k\left(\frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}}\right)^T$$

where $U(\boldsymbol{q})$ is a differentiable objective function: we try to locally maximize $U(\boldsymbol{q})$ while executing the task.

Examples for the objective function might be:

- manipulability index
- distance from joint limits
- distance from obstacles

2.4 Explain what we mean with cyclic or repeatable kinematic inversion method. Cite one such method.

A cyclic or repeatable method guarantees that a closed trajectory in the task space is mapped into a closed trajectory in joint space. An example of such methods is the extended (or augmented) Jacobian method.

Exercise 3

Consider the planning of a trajectory for the orientation of the end-effector in the operational space.

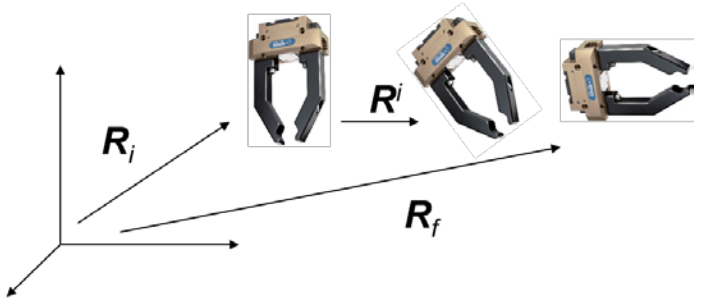
3.1 Write the formula for the interpolation of a set of three Euler angles.

The formula is:

$$\phi(s) = \phi_i + \frac{s}{\|\phi_f - \phi_i\|}(\phi_f - \phi_i)$$

where ϕ_i are the initial values of the Euler angles, ϕ_f are the final values, while s is a natural coordinate ranging from 0 to $\| \phi_f - \phi_i \|$.

3.2 Making reference now to the following picture, explain how the orientation can be planned using the axis-angle representation.



Define with $\boldsymbol{R}^i(t)$ the matrix that describes the transition from $\boldsymbol{R}_i$ (initial orientation) to $\boldsymbol{R}_f$ (final orientation).

Then:

$$\boldsymbol{R}^i(0) = \boldsymbol{I}, \quad \boldsymbol{R}^i(t_f) = \boldsymbol{R}^i_f$$

Matrix $\boldsymbol{R}^i(t)$ can be interpreted with the axis-angle representation as $\boldsymbol{R}^i(\vartheta(t), \boldsymbol{r})$, where:

- $\boldsymbol{r}$ is constant and can be computed from the elements of $\boldsymbol{R}^i_f$
- $\vartheta(t)$ can be made variable with time, through a suitable timing law, with $\vartheta(0)=0$, $\vartheta(t_f)=\vartheta_f$

In order to characterize at each time t the orientation in the base frame it is then enough to compute:

$$\boldsymbol{R}(t) = \boldsymbol{R}_i \boldsymbol{R}^i(t)$$

3.3 Explain what is the advantage in using the axis-angle representation compared to the interpolation of a set of three Euler angles.

The interpolation of the Euler angles suffers from poor predictability and understanding of the intermediate orientation. This problem is mitigated with the use of the axis-angle representation.

3.4 Suppose that the angle in the axis/angle representation has to change from 0 to 1 (rad) in 2 seconds. Plan a cubic trajectory for the angle.

We need to find the coefficients of the polynomial:

$$\vartheta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

with the boundary conditions:

$$\vartheta(0) = 0 \quad \vartheta(2) = 1$$
$$\dot{\vartheta}(0) = 0 \quad \dot{\vartheta}(2) = 0$$

From the conditions at $t = 0$, we easily obtain $a_0 = a_1 = 0$. From the conditions at $t = 2$ we obtain the linear system:

$$\begin{cases} 4a_2 + 8a_3 = 1 \\ 4a_2 + 12a_3 = 0 \end{cases} \Rightarrow \begin{cases} a_2 = 3/4 \\ a_3 = -1/4 \end{cases}$$

Then the expression of the cubic polynomial is:

$$\vartheta(t) = \frac{3t^2 - t^3}{4}$$

Exercise 4

Consider the independent joint control of a manipulator.

4.1 Explain what is meant with "decentralized model" and how such approximation is used in the independent joint control.

The "decentralized model" approximation means that all the coupling and nonlinear terms of the dynamic model of the manipulator are ignored and only the inertias at the joints, along with the effects of the inertias of the motors, are considered.

Leveraging on this decentralized model, in the independent joint control the control system is articulated in n SISO control loops, where the dynamic coupling effects between joints are dealt with as disturbances. The single control problems are addressed as control of a servomechanism.

4.2 In a two-link planar manipulator with rotational joints, the inertia matrix takes the following expression:

$$\boldsymbol{B}(\boldsymbol{q}) = \begin{bmatrix} m_1 l_1^2 + I_1 + m_2\left(a_1^2 + l_2^2 + 2a_1 l_2 \cos(\vartheta_2)\right) + I_2 & m_2\left(l_2^2 + a_1 l_2 \cos(\vartheta_2)\right) + I_2 \\ m_2\left(l_2^2 + a_1 l_2 \cos(\vartheta_2)\right) + I_2 & m_2 l_2^2 + I_2 \end{bmatrix}$$

Explain what is the approximate form of such matrix that is used in the decentralized model.

In the decentralized model the off-diagonal terms of the inertia matrix are ignored, while an average value is taken for each of the diagonal terms.

In this specific case this can be done for example taking a value of ϑ_2 such that $\cos(\vartheta_2)$ is zero.

We thus obtain:

$$\overline{\boldsymbol{B}} = \begin{bmatrix} m_1 l_1^2 + I_1 + m_2\left(a_1^2 + l_2^2\right) + I_2 & 0 \\ 0 & m_2 l_2^2 + I_2 \end{bmatrix}$$

4.3 Assume now that m_2 = 50, l_2 = 0.5, I_2 = 10. For the second joint of the planar two-link manipulator, where the following actuation system is used:

motor inertia J_m = 0.005

reduction ratio n = 100

tune a PI speed controller so as to achieve a crossover frequency ω_{cv} = 200 rad/s.

In the servomechanism problem for the second joint, the load inertia is:

$$J_l = m_2 l_2^2 + I_2 = 22.5$$

The load inertia reflected at the motor axis is then:

$$J_{lr} = \frac{J_l}{n^2} = 0.00225$$

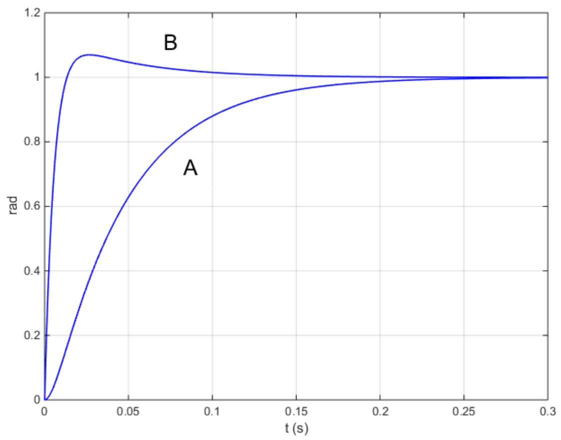
The gain of the controller is:

$$K_{pv} = \frac{\omega_{cv}}{\mu} = (J_m + J_{lr})\omega_{cv} = 0.00725 * 200 = 1.45$$

The integral time of the controller can be set as:

$$T_{iv} = \frac{10}{\omega_{cv}} = \frac{10}{200} = 0.05$$

4.4 Tune a proportional position controller in order to achieve a crossover frequency ω_{cp} = 20 rad/s. Suppose then that a step response for the closed loop position control system is obtained. Explain which one, out of the two plots reported here, is obtained with speed feedforward and which one is without.



In order to tune the controller it is enough to set:

$$K_{pp} = \omega_{cp} = 20$$

Speed feedforward is used to speed up the response of the position, achieving a bandwidth comparable to the one of the speed control loop. Thus response A refers to the case without speed feedforward, response B with.