Control of industrial robots

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- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
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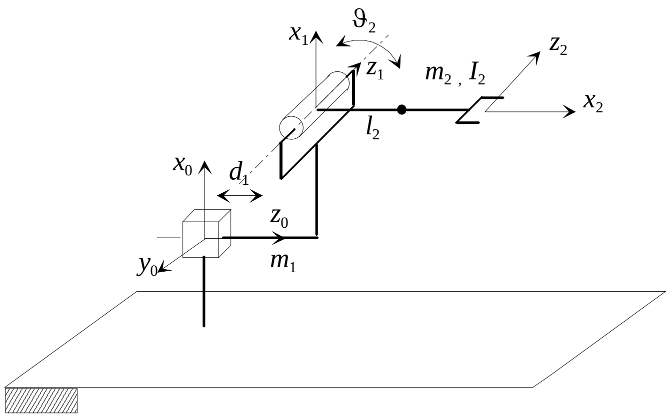
Exercise 1

Consider the manipulator sketched in the picture:



1.1 Find the expression of the inertia matrix of the manipulator[1].

Denavit-Hartenberg frames can be defined as sketched in the picture.

Computations of the Jacobians:

Link 1

$$\boldsymbol{J}_P^{(l_1)} = \begin{bmatrix} \boldsymbol{j}_{P1}^{(l_1)} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_0 & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

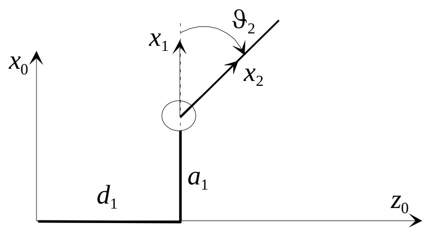
Link 2

$$\boldsymbol{J}_P^{(l_2)} = \begin{bmatrix} \boldsymbol{j}_{P1}^{(l_2)} & \boldsymbol{j}_{P2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_0 & \boldsymbol{z}_1 \times (\boldsymbol{p}_{l2} - \boldsymbol{p}_1) \end{bmatrix} = \begin{bmatrix} 0 & -l_2 s_2 \\ 0 & 0 \\ 1 & l_2 c_2 \end{bmatrix}$$

$$\boldsymbol{J}_O^{(l_2)} = \begin{bmatrix} \boldsymbol{j}_{O1}^{(l_1)} & \boldsymbol{j}_{O2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{z}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

Auxiliary vectors for the above computations:

$$\boldsymbol{p}_{l2} = \begin{bmatrix} a_1 + l_2 c_2 \\ 0 \\ d_1 + l_2 s_2 \end{bmatrix}, \quad \boldsymbol{p}_1 = \begin{bmatrix} a_1 \\ 0 \\ d_1 \end{bmatrix}, \quad \boldsymbol{z}_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$



Inertia matrix:

[1] The cross product between vectors $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

$$\boldsymbol{B}(\boldsymbol{q}) = m_1 \boldsymbol{J}_P^{(l_1)T} \boldsymbol{J}_P^{(l_1)} + m_2 \boldsymbol{J}_P^{(l_2)T} \boldsymbol{J}_P^{(l_2)} + I_2 \boldsymbol{J}_O^{(l_2)T} \boldsymbol{J}_O^{(l_2)} = m_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} 1 & l_2 c_2 \\ l_2 c_2 & l_2^2 \end{bmatrix} + I_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$

with:

$$b_{11} = m_1 + m_2$$

$$b_{12} = m_2 l_2 c_2$$

$$b_{22} = m_2 l_2^2 + I_2$$

1.2 Write the expression of the gravitational terms for this manipulator.

The gravity acceleration vector is:

$$\boldsymbol{g}_0 = \begin{bmatrix} -g \\ 0 \\ 0 \end{bmatrix}$$

Then the gravitational torques are:

$$\boldsymbol{g} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix},$$

where:

$$g_1 = -m_1 \boldsymbol{g}_0^T \boldsymbol{j}_{P1}^{(l_1)} - m_2 \boldsymbol{g}_0^T \boldsymbol{j}_{P1}^{(l_2)} = -m_1 \boldsymbol{g}_0^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - m_2 \boldsymbol{g}_0^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$g_2 = -m_1 \boldsymbol{g}_0^T \boldsymbol{j}_{P2}^{(l_1)} - m_2 \boldsymbol{g}_0^T \boldsymbol{j}_{P2}^{(l_2)} = -m_2 \boldsymbol{g}_0^T \begin{bmatrix} -l_2 s_2 \\ 0 \\ l_2 c_2 \end{bmatrix} = -m_2 g l_2 s_2$$

1.3 Write the expression of a "PD + gravity compensation" control law in the joint space for this specific manipulator.

The vector equation of the control law is:

$$\boldsymbol{\tau} = \boldsymbol{K}_P (\boldsymbol{q}_d - \boldsymbol{q}) - \boldsymbol{K}_D \dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q})$$

and corresponds, for the given manipulator, to the following two equations:

$$\begin{cases} \tau_1 = K_{P1}(q_{d1} - q_1) - K_{D1}\dot{q}_1 \\ \tau_2 = K_{P2}(q_{d2} - q_2) - K_{D2}\dot{q}_2 + m_2 g l_2 s_2 \end{cases}$$

1.4 Write the expression of a "PD + gravity compensation" control law in the operational space for the generic manipulator.

$$\boldsymbol{\tau} = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{J}_A^T(\boldsymbol{q}) \boldsymbol{K}_P \tilde{\boldsymbol{x}} - \boldsymbol{J}_A^T(\boldsymbol{q}) \boldsymbol{K}_D \boldsymbol{J}_A(\boldsymbol{q}) \dot{\boldsymbol{q}}$$

Exercise 2

Consider a kinematically redundant manipulator.

2.1 Explain what the “null-space motions” are.

Null space motions are motions in the joint space that do not change the task variables (usually end effector position and orientation).

2.2 Write an expression for the null-space motions, explaining the meaning of each symbol used.

Given a set of task variables $\boldsymbol{r}$, such that:

$$\dot{\boldsymbol{r}} = \boldsymbol{J}\dot{\boldsymbol{q}}$$

where $\boldsymbol{J}$ is the Jacobian matrix, the general expression of the solution of the inverse kinematics is:

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\#}\dot{\boldsymbol{r}} + \boldsymbol{P}\dot{\boldsymbol{q}}_0$$

where $\boldsymbol{J}^{\#}$ is the pseudo-inverse of the Jacobian, defined as:

$$\boldsymbol{J}^{\#} = \boldsymbol{J}^T\left(\boldsymbol{J}\boldsymbol{J}^T\right)^{-1}$$

The term $\boldsymbol{P}\dot{\boldsymbol{q}}_0$ defines the null-space motions.

$\boldsymbol{P}$ is a matrix that projects a generic joint velocity $\dot{\boldsymbol{q}}_0$ in the null space of the Jacobian and takes the expression:

$$\boldsymbol{P} = \boldsymbol{I}_n - \boldsymbol{J}^{\#}\boldsymbol{J}$$

2.3 Consider now motion planning of the end-effector position. Select as an initial point $\boldsymbol{p}_i$ = [1, 0, 1] and as a final point $\boldsymbol{p}_f$ = [3, 2, 2]. Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.

The general expression of the segment is:

$$\boldsymbol{p}(s) = \boldsymbol{p}_i + \frac{s}{\left\|\boldsymbol{p}_f - \boldsymbol{p}_i\right\|}\left(\boldsymbol{p}_f - \boldsymbol{p}_i\right)$$

Since:

$$\boldsymbol{p}_f - \boldsymbol{p}_i = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \left\|\boldsymbol{p}_f - \boldsymbol{p}_i\right\| = 3$$

it is:

$$\boldsymbol{p}(s) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \frac{s}{3}\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

2.4 Assume that the maximum linear velocity and the maximum linear acceleration of the end-effector are given by $v_{\max} = 2$ m/s and $a_{\max} = 3$ m/s^2, respectively. Assuming a trapezoidal velocity profile, find the minimum travelling time for the trajectory.

The total displacement is h = 3.

Since the following inequality holds true:

$$h > \frac{v_{\max}^{2}}{a_{\max}} = \frac{4}{3}$$

the minimum positioning time is:

$$T = \frac{h}{v_{\max}} + \frac{v_{\max}}{a_{\max}} = \frac{3}{2} + \frac{2}{3} = \frac{13}{6}$$

Exercise 3

Consider the decentralized control of a manipulator.

3.1 Explain what is meant with "independent joint control".

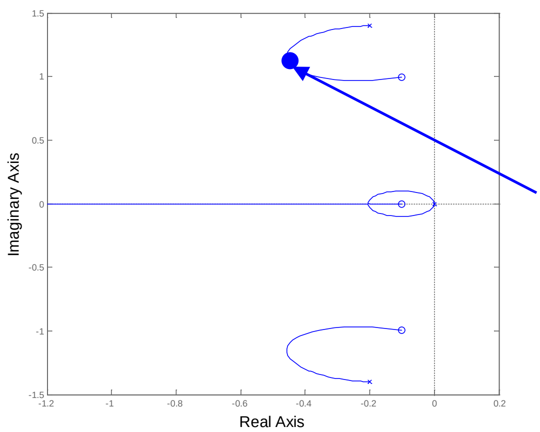
In the independent joint control the control system is articulated in n SISO control loops, ignoring the dynamic coupling effects between joints, which are dealt with as disturbances. The single control problems are addressed as control of a servomechanism.

3.2 What is the property of industrial robots upon which such method heavily relies? Why?

The method heavily relies on the large values of the reduction ratios adopted in robotics. The effects of the nonlinearities and of the couplings between joints scale down with the reduction ratios: the larger these are, the more reliable is the assumption of a linear and decoupled system which is enforced when using the independent joint control.

3.3 Assume now that the joints are affected by flexibility: sketch the root locus of the speed control and explain what is the graphical method to tune the speed controller based on such locus.

The root locus takes this shape:



The graphical method consists in selecting the gain of the speed controller in such a way that the damping of the complex and conjugate poles is maximized (see the picture).

This is usually obtained tuning the nominal (i.e. computed on the rigid model) bandwidth of the speed control loop as:

$$\omega_{cv} \approx 0.7\omega_z$$

where ω_z is (an estimate of) the antiresonance frequency (natural frequency of the zeros).

3.4 Assume now the following values for the physical parameters of one of the joints :

J_l = 9 Kg m^2, ρ = 2, n = 30, K_{el} = 400.

Tune a PI speed controller for this servomechanism.

The load inertia reflected at the motor axis is:

$$J_{lr} = \frac{J_l}{n^2} = 0.01$$

The motor inertia is:

$$J_m = \frac{J_{lr}}{\rho} = 0.005$$

The gain of the system is:

$$\mu = \frac{1}{J_m + J_{lr}} = 20$$

The antiresonance frequency:

$$\omega_z = \sqrt{\frac{K_{el}}{J_{lr}}} = 200$$

The nominal crossover frequency is then:

$$\omega_{cv} = 0.7\omega_z = 140$$

We use the following tuning formula:

$$K_{pv}\mu = \omega_{cv}$$

Then:

$$K_{pv} = \frac{\omega_{cv}}{\mu} = (J_m + J_{lr})\omega_{cv} = 0.015 * 140 = 2.1$$

The integral time of the controller is:

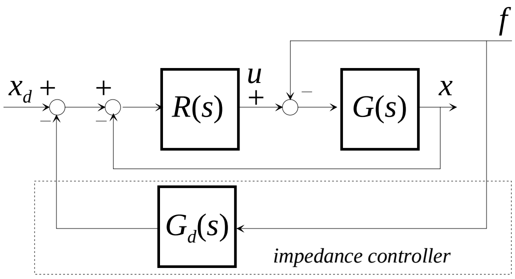
$$T_{iv} = \frac{10}{\omega_z} = 0.05$$

Exercise 4

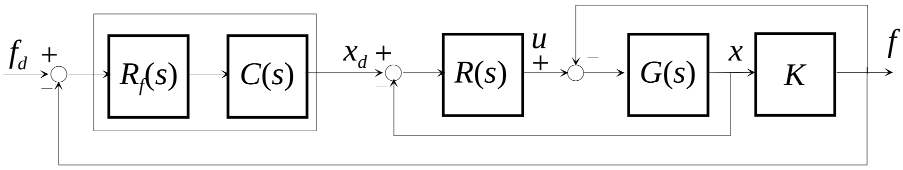
4.1 Explain the difference between an impedance control and a force control.

With the impedance control the goal is to assign a prescribed dynamic relation between interaction forces and position errors. With the force control we want that in the directions constrained by the environment a force/torque with a specified value be established.

4.2 Sketch the block diagram of an implicit impedance control scheme for a single d.o.f. system.

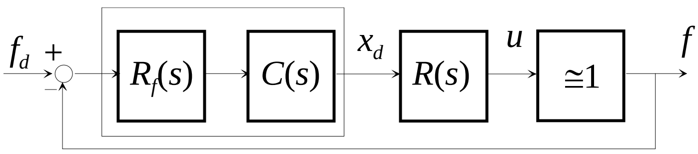


4.3 Sketch the block diagram of an implicit force control scheme for a single d.o.f. system, in contact with an environment.



4.4 Assume that the contact established with the environment is practically rigid and that the system of item 4.3 is position controlled with a PID, whose gains are K_P = 60, K_I = 10, K_D = 50. Find the expression of the force controller, tuned to achieve a bandwidth of 30 rad/s.

If contact is rigid, the previous block diagram can be simplified as:



The transfer function $C(s)$ partially compensates for the transfer function of the PID controller:

$$C(s) = \frac{1}{K_D s^2 + K_P s + K_I} = \frac{1}{50s^2 + 60s + 10}$$

Controller $R_f(s)$ is a PI:

$$R_f(s) = K_{pf} + \frac{K_{if}}{s}$$

which means that the loop transfer function is:

$$L_f(s) = \frac{sK_{pf} + K_{if}}{s^2}$$

Referring to the Bode diagram qualitatively sketched here, we can tune the controller as follows:

$$K_{pf} \approx \omega_{cf} = 30$$

$$\frac{K_{if}}{K_{pf}} = 6 \quad \Rightarrow \quad K_{if} = 180$$