

# Control of industrial robots

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## Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

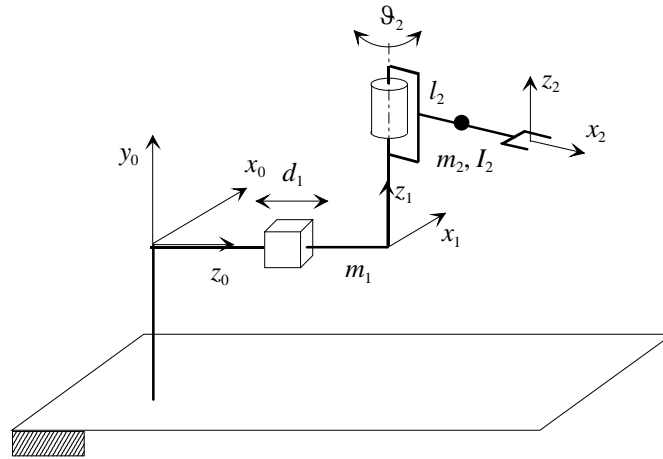
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**Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient**

**Exercise 1**

Consider the manipulator sketched in the picture:



**1.1** Find the expression of the inertia matrix of the manipulator<sup>1</sup>.

Denavit-Hartenberg frames can be defined as sketched in the picture.

Computations of the Jacobians:

*Link 1*

$$J_P^{(l_1)} = [j_{P1}^{(l_1)} \quad \mathbf{0}] = [z_0 \quad \mathbf{0}] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

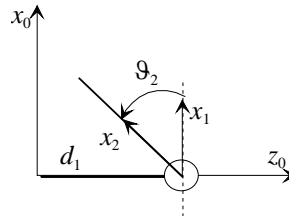
*Link 2*

$$J_P^{(l_2)} = [j_{P1}^{(l_2)} \quad j_{P2}^{(l_2)}] = [z_0 \quad z_1 \times (p_{l_2} - p_1)] = \begin{bmatrix} 0 & -l_2 s_2 \\ 0 & 0 \\ 1 & -l_2 c_2 \end{bmatrix}$$

$$J_O^{(l_2)} = [j_{O1}^{(l_2)} \quad j_{O2}^{(l_2)}] = [\mathbf{0} \quad z_1] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Auxiliary vectors for the above computations:

$$p_{l_2} = \begin{bmatrix} l_2 c_2 \\ 0 \\ d_1 - l_2 s_2 \end{bmatrix}, \quad p_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Inertia matrix:

<sup>1</sup> The cross product between vectors  $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$  and  $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$  is  $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

$$\begin{aligned} \mathbf{B}(\mathbf{q}) &= m_1 \mathbf{J}_P^{(l_1)T} \mathbf{J}_P^{(l_1)} + m_2 \mathbf{J}_P^{(l_2)T} \mathbf{J}_P^{(l_2)} + I_2 \mathbf{J}_O^{(l_2)T} \mathbf{J}_O^{(l_2)} = m_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} 1 & -l_2 c_2 \\ -l_2 c_2 & l_2^2 \end{bmatrix} + I_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \\ &= \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \end{aligned}$$

with:

$$b_{11} = m_1 + m_2$$

$$b_{12} = -m_2 l_2 c_2$$

$$b_{22} = m_2 l_2^2 + I_2$$

### 1.2 Compute the expression of the Coriolis and centrifugal terms for this manipulator<sup>2</sup>.

The only derivative in the Christoffel symbols which is different from zero is:

$$\frac{\partial b_{12}}{\partial q_2} = l_2 m_2 s_2$$

Therefore:

$$c_{111} = 0$$

$$c_{211} = 0$$

$$c_{112} = c_{121} = 0$$

$$c_{212} = c_{221} = \frac{1}{2} \left( \frac{\partial b_{12}}{\partial q_2} - \frac{\partial b_{12}}{\partial q_2} \right) = 0$$

$$c_{122} = \frac{1}{2} \left( \frac{\partial b_{12}}{\partial q_2} + \frac{\partial b_{12}}{\partial q_2} \right) = l_2 m_2 s_2 \quad c_{222} = 0$$

The matrix of the Coriolis and centrifugal terms is thus:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \quad \begin{aligned} c_{11} &= c_{111} \dot{q}_1 + c_{112} \dot{q}_2 = 0 \\ c_{12} &= c_{121} \dot{q}_1 + c_{122} \dot{q}_2 = l_2 m_2 s_2 \dot{\vartheta}_2 \\ c_{21} &= c_{211} \dot{q}_1 + c_{212} \dot{q}_2 = 0 \\ c_{22} &= c_{221} \dot{q}_1 + c_{222} \dot{q}_2 = 0 \end{aligned}$$

### 1.3 Check that matrix $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew-symmetric.

Matrix  $\mathbf{N}$  can be written as

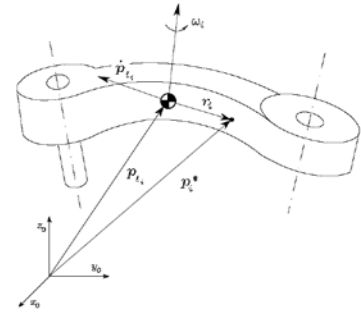
$$\mathbf{N} = \dot{\mathbf{B}} - 2\mathbf{C} = \begin{bmatrix} 0 & l_2 m_2 s_2 \dot{\vartheta}_2 \\ l_2 m_2 s_2 \dot{\vartheta}_2 & 0 \end{bmatrix} - 2 \begin{bmatrix} 0 & l_2 m_2 s_2 \dot{\vartheta}_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -l_2 m_2 s_2 \dot{\vartheta}_2 \\ l_2 m_2 s_2 \dot{\vartheta}_2 & 0 \end{bmatrix}$$

and is indeed skew-symmetric.

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<sup>2</sup> The expression of the Christoffel symbols is:  $c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

- 1.4 Consider a single link, as sketched in the picture. Without going through the derivation, write the expression of the kinetic energy of the link, specifying the meaning of all the symbols used.



The expression of the kinetic energy of the link is:

$$T_i = \frac{1}{2} m_i \dot{\mathbf{p}}_i^T \dot{\mathbf{p}}_i + \frac{1}{2} \boldsymbol{\omega}_i^T \mathbf{I}_i \boldsymbol{\omega}_i$$

where:

$m_i$  is the mass,  $\mathbf{I}_i$  is the inertia tensor,  $\dot{\mathbf{p}}_i$  is the linear velocity of the center of mass, and  $\boldsymbol{\omega}_i$  is the angular velocity vector.

## Exercise 2

- 2.1 Using any motion programming language (for example COMAU PDL2) write the instruction to command a motion on a straight line towards a certain final position. Explain what options are usually available to command a motion other than along a straight line.

Using COMAU PDL2, the instruction takes this form:

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MOVE LINEAR TO P1
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where P1 is a target position.

Other possibilities are a motion along an arc (MOVE CIRCULAR) or a motion in joint space (MOVE JOINT).

- 2.2 Write the parametric expression (in terms of a natural coordinate) of a segment in space, used for planning a linear path.

The parametric expression of a segment is:

$$\mathbf{p}(s) = \mathbf{p}_i + \frac{s}{\|\mathbf{p}_f - \mathbf{p}_i\|} (\mathbf{p}_f - \mathbf{p}_i)$$

where  $\mathbf{p}_i$  and  $\mathbf{p}_f$  are the initial and final positions, respectively.

- 2.3 Prove that, in the general case, the absolute value of the time derivative of the natural coordinate  $s$  is the norm of the linear velocity of the end-effector.

Given a generic path parametrized in the natural coordinate  $\mathbf{p}(s)$ , the time derivative of  $\mathbf{p}$  is:

$$\dot{\mathbf{p}} = \dot{s} \frac{d\mathbf{p}}{ds} = \dot{s} \mathbf{t}$$

where  $\mathbf{t}$  is the tangent unit vector.

Therefore the absolute value of  $\dot{s}$  is the norm of vector  $\dot{\mathbf{p}}$

- 2.4 Assume now that the length of the segment to cover is 0.5 m and that the maximum linear velocity of the end effector is 1.5 m/s. Compute the minimum positioning time, if a cubic dependence on time of the natural coordinate is used.

The parametric form of a cubic trajectory is:

$$s(t) = s_i + h\sigma(\tau)$$

where the parameter  $\sigma$  takes the form:

$$\sigma(\tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3$$

The maximum value of the derivative with respect to time of  $s$  is related to the travel time  $T$  and to the total displacement  $h$  by the formula:

$$\dot{s}_{\max} = \frac{3h}{2T}$$

Since the derivative of  $s$  is the linear velocity of the end-effector, we have the inequality:

$$\frac{3h}{2T} \leq v_{\max} = 1.5 \Rightarrow T \geq \frac{3 \cdot 0.5}{2 \cdot 1.5} = 0.5$$

The minimum positioning time is then 0.5.

### Exercise 3

Consider a kinematically redundant manipulator.

**3.1** Explain what the “null-space motions” are, and write an expression for the null-space motions, explaining the meaning of each symbol used.

Null-space motions are motion in the joint space that do not yield any motion of the task variables. Given some task variables  $r$ , the general expression of the joint velocities is:

$$\dot{q} = J^{\#} \dot{r} + (I_n - J^{\#} J) \dot{q}_0$$

where  $J$  is the Jacobian of the task variables,  $J^{\#}$  is its pseudo inverse matrix, and  $\dot{q}_0$  is a vector of joint velocities to be projected in the null space of the Jacobian.

**3.2** Consider a standard six degrees of freedom robot manipulator with rotational joints: specify a task for which this robot is redundant and the size of the null space of the Jacobian for such task.

A six d.o.f. robot manipulator is redundant for the task of keeping a certain position at the end effector with arbitrary orientation. Since this task has dimension 3, the null space of the Jacobian has dimension  $6 - 3 = 3$ .

**3.3** Explain how the null-space motions can be used to satisfy secondary criteria, making at least two examples of such criteria.

The joint space velocities  $\dot{q}_0$  can be shaped as proportional to the gradient of a potential function:

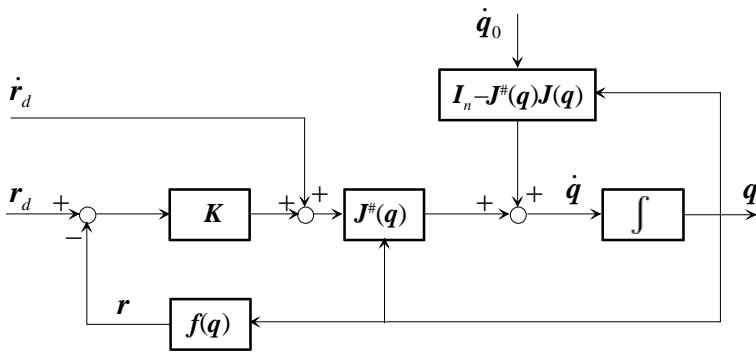
$$\dot{q}_0 = k \left( \frac{\partial U(q)}{\partial q} \right)^T$$

with the aim to approximately optimize such function.

Examples for the potential function can be the manipulability measure (to stay away from singularities), the distance to the joint limits and the minimum distance from the closest obstacle.

**3.4** Consider now a closed-loop kinematic control (i.e. an inverse kinematics scheme) for a redundant robot: sketch the block diagram of the controller.

The block diagram takes this shape:



**Exercise 4**

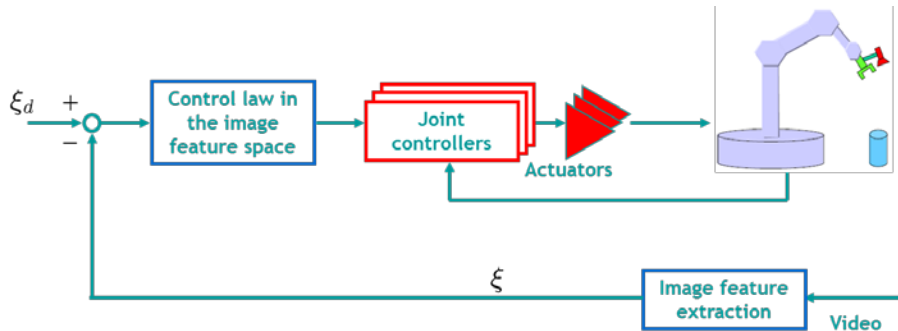
Consider the control of a manipulator with vision sensors.

**4.1** Explain what we mean with “image-based” controller.

In an image-based control scheme the error that drives the controller is computed directly in terms of errors between desired and actual image features.

**4.2** Sketch the block diagram of an image-based look-and-move controller. Briefly explain what is the advantage of using a look-and-move configuration.

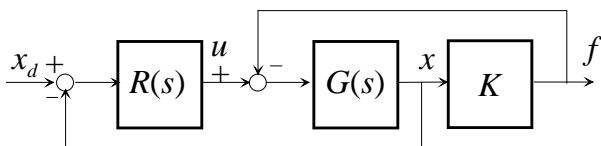
The block diagram can be sketched as follows.



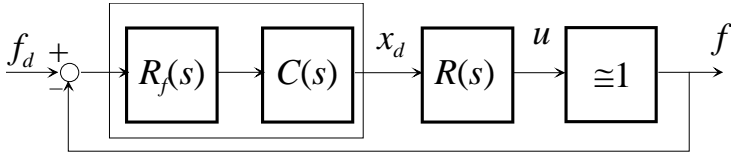
In a look-and-move configuration the visual controller is closed outside the industrial position controllers and can rely on their high bandwidth, so that the position controlled robot can be seen as an ideal positioner.

**4.3** Consider now a robot equipped with force sensors. For a single degree of freedom mechanism, sketch the block diagram of an implicit force controller in case of contact with a stiff environment. Briefly explain what is the advantage of using an implicit configuration.

The block diagram of the position controlled mechanical system in contact with the environment is:



Assuming a stiff environment ( $K \rightarrow \infty$ ) and closing a loop on the force feedback, we have:



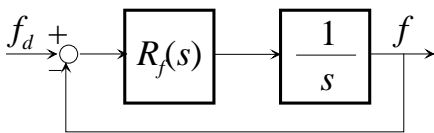
where the force controller partially compensates for the dynamics of the position controller.

The advantage of using an implicit controller is to rely on the industrial position controllers that are left unchanged.

**4.4** Assuming that the dynamics of the position controller are correctly compensated, design the force controller in such a way to obtain a bandwidth of 10 rad/s.

The dynamics of the position controller can be compensated except for the integrator.

The closed loop system then becomes:



Taking a PI as a force controller:

$$R_f(s) = K_{pf} + \frac{K_{if}}{s}$$

the loop transfer function is:

$$L_f(s) = \frac{sK_{pf} + K_{if}}{s^2}$$

As it is clear from the Bode diagram, a suitable choice of the controller gain can be:

$$K_{pf} = \omega_{cf} = 10$$

$$\frac{K_{if}}{K_{pf}} = 2 \Rightarrow K_{if} = 20$$

