

# Control of Industrial Robots

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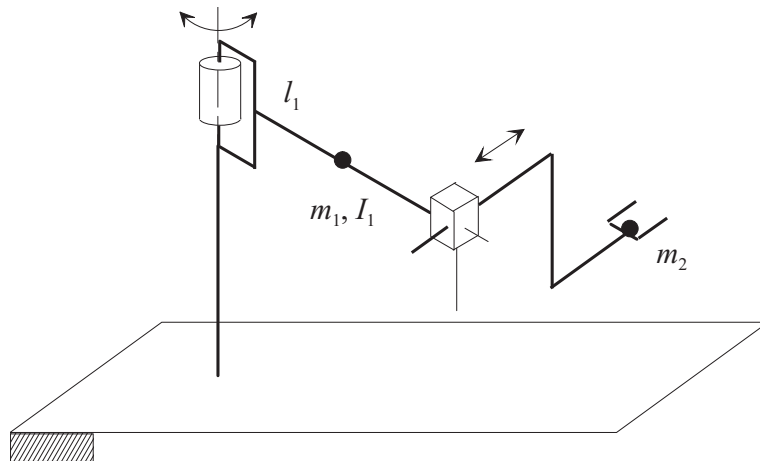
## Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.



## EXERCISE 1

1. Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



Find the expression of the inertia matrix  $\mathbf{B}(\mathbf{q})$  of the manipulator.

2. Compute the matrix  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  of the Coriolis and centrifugal terms<sup>1</sup> for this manipulator.

3. Check that matrix  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is skew symmetric.

4. Cite one case in robotics where the property that matrix  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$  is skew symmetric is used.

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<sup>1</sup>The general expression of the Christoffel symbols is  $c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

## EXERCISE 2

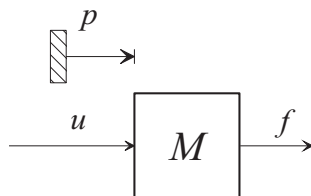
1. Consider the generation of a position trajectory in the Cartesian space. Select as an initial point  $\mathbf{p}_i = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$  and as a final point  $\mathbf{p}_f = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$ . Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.
2. Assume a travel time  $T = 2s$ . Design a trajectory, which covers the path determined in the previous step, using a cubic dependence on time.

3. Compute the maximum linear velocity of the end effector along the trajectory designed in the previous step. Check whether this maximum value exceeds a maximum admissible velocity of 2 m/s. In case, explain (without going through the computations) how you would modify the trajectory generation.
4. Suppose that the manipulator is kinematically redundant for the task of end effector positioning. Write the expression of the inverse kinematics based on the *weighted* pseudo-inverse matrix and explain what is the optimization problem solved with this approach. Also explain what might be the reason to use the weighted pseudo-inverse instead of the standard pseudo-inverse matrix.

### EXERCISE 3

1. Define the mechanical impedance of a system and explain what is the purpose of an impedance controller.

2. Consider now a simple mass as in this picture:



Write the expression of an (explicit) impedance controller that can assign a prescribed and complete impedance relation.

3. Still making reference to a single degree of freedom mechanism, sketch the block diagram of an admittance controller. What is the assumption that must be enforced on the motion control system in order to claim that the prescribed impedance is actually achieved?

4. The admittance controller can be used to implement one of the possible collaborative modes between the robot and the human. Explain what is this mode and cite also the other collaborative modes allowed by the safety standards.