# Control of industrial robots

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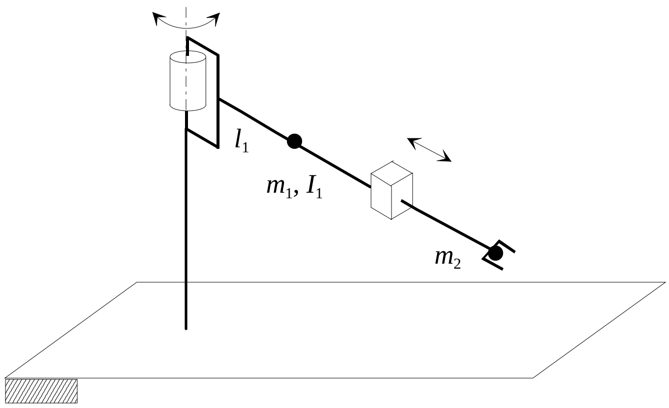
**Warnings:**

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

**Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient**

**Exercise 1**

Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector




**1.1** Find the expression of the inertia matrix of the manipulator.

**1.2** Compute the matrix $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ of the Coriolis and centrifugal terms [1] for this manipulator

**1.3** Write the complete dynamic model for this manipulator

**1.4** Show that the model obtained in the previous step is linear with respect to a set of dynamic parameters. Is it possible to identify through experiments the mass of the first link ($m_1$)?

[1] The general expression of the Christoffel symbols is: $c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$

**Exercise 2**

**2.1** Explain what is meant with kinematic scaling of a trajectory and write the general expression of a trajectory in the form (parameterized) which is used in such scaling.

**2.2** The parametric form of a harmonic trajectory is given by:

$$\sigma(\tau) = \frac{1}{2}\left(1 - \cos(\pi\tau)\right)$$

Find the expressions of the maximum velocity and maximum acceleration for such trajectory in terms of the positioning time *T* and the total displacement *h*.

**2.3** Consider the design of a harmonic trajectory from $q_i$ = 10 to $q_f$ = 30, with $\dot{q}_{\max} = 20$, $\ddot{q}_{\max} = 10$. Find the minimum positioning time.

**2.4** For the harmonic trajectory computed in this exercise, sketch the plot of the speed $\dot{q}(t)$, assuming that the trajectory starts at time $t_i$=0.

**Exercise 3**

**3.1** Write the expression of a "PD plus gravity compensation" control law in joint space.

**3.2** Write the expression of the Lyapunov function used to prove the stability property of such control scheme and explain why this Lyapunov function is positive definite.

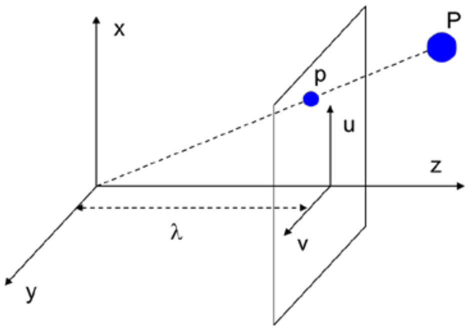
**3.3** Compute the derivative of the Lyapunov function along the trajectory of the system, when using the PD plus gravity compensation control law. Explain whether this derivative is definite, semidefinite or not definite in sign.

**3.4** Explain what is the theoretical result that can be achieved with the PD plus gravity compensation control law and how such result can be proven.

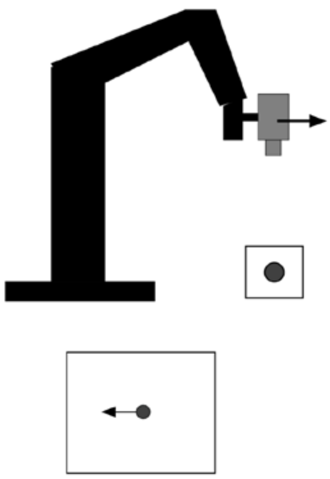
**Exercise 4**

Consider the control of a manipulator with vision sensors.

**4.1** Explain what is the “perspective projection” method and, making reference to the following picture, write the related formulas.




**4.2** Making reference to the following picture, explain what is the interaction matrix in the context of visual control, specifying the size of such matrix.



**4.3** Specify whether the interaction matrix presents a null-space and comment on possible physical interpretations of such null-space.

**4.4** Explain what is the image Jacobian and what is its relation with the interaction matrix.