

# Control of industrial robots

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## Warnings:

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

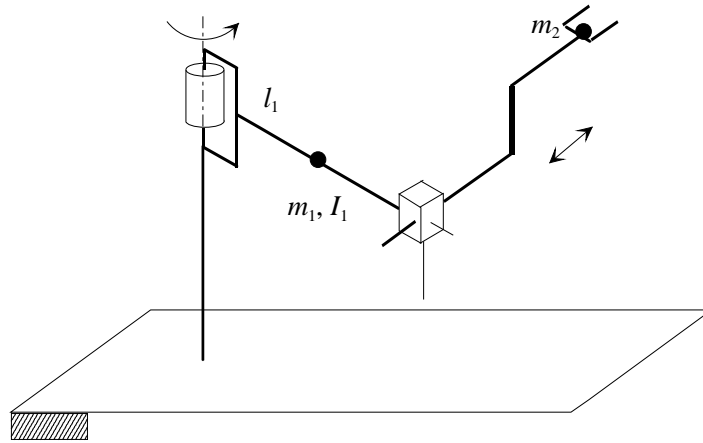
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**Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient**

**Exercise 1**

Consider the manipulator sketched in the picture, where the mass of the second link is assumed to be concentrated at the end-effector:



**1.1** Find the expression of the inertia matrix  $\mathbf{B}(\mathbf{q})$  of the manipulator.

1.2 Compute the matrix  $C(\mathbf{q}, \dot{\mathbf{q}})$  of the Coriolis and centrifugal terms <sup>1</sup> for this manipulator.

1.3 Show that matrix  $N(\mathbf{q}, \dot{\mathbf{q}}) = \dot{\mathbf{B}}(\mathbf{q}) - 2C(\mathbf{q}, \dot{\mathbf{q}})$  is skew symmetric.

1.4 Compute the expression of the derivative of the kinetic energy  $T(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}(\mathbf{q}) \dot{\mathbf{q}}$  (notice that no gravitational load acts on the manipulator).

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<sup>1</sup> The general expression of the Christoffel symbols is:  $c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i} \right)$

**Exercise 2**

Consider a P/PI control system for a rigid servomechanism.

Assume the following values for the physical parameters:

$$J_m = 0.02 \text{ Kg } m^2$$

$$D_m \cong 0$$

$$\rho = 3 \text{ (inertia ratio)}$$

**2.1** Design a speed PI controller in such a way to obtain a crossover frequency  $\omega_{ev} \cong 100 \text{ rad/s}$

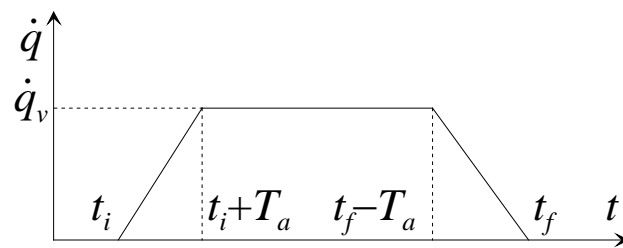
**2.2** Design a P position controller in such a way to obtain a crossover frequency  $\omega_{cp} \cong 20 \text{ rad/s}$ .

**2.3** Explain what is the “speed feedforward” and for what reason it is used in motion control systems.

- 2.4 Assume now that a (unitary gain) speed feedforward is used and that speed measurement is obtained through differentiation of the position signal. Show that the resulting controller is equivalent to a PID on the position error.

### Exercise 3

Consider a trajectory characterized by the trapezoidal velocity profile sketched in the picture:



- 3.1 Set  $t_i = 0$ ,  $t_f = 3$ ,  $\dot{q}_v = 2$ , and find the acceleration time  $T_a$  in such a way that the distance covered is  $h = 4$ .

- 3.2 Assume now that the maximum allowable acceleration is the one determined at the previous step of this exercise for the initial acceleration phase. Assume however that the maximum speed be limited  $\dot{q}_{\max} = 1$ . Find the minimum positioning time in such situation.

**3.3** Consider a multi degree of freedom manipulator. Briefly describe a method to assign joint space trajectories in such a way to minimize positioning time and to have all the joints complete their tasks simultaneously.

**3.4** Assume now that some intermediate points are given and the whole trajectory has to be covered connecting trajectories with trapezoidal velocity profile. Describe a method that can be used to avoid that motion is stopped at each intermediate point.

**Exercise 4**

**4.1** Explain the difference between an implicit and an explicit force controller.

**4.2** What are the potential advantages of an implicit configuration?

**4.3** Sketch the block diagram of an implicit force control scheme for a single d.o.f. system, in contact with an environment.

**4.4** Assume that the contact established with the environment is practically rigid and that the system of item 4.3 is position controlled with a PID, whose gains are  $K_P = 30$ ,  $K_I = 20$ ,  $K_D = 10$ . Find the expression of the force controller, tuned to achieve a bandwidth of 30 rad/s.