# Control of industrial robots

(Prof. Rocco)

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Name: ..................................................

University ID number:...............................

Signature:.......................................

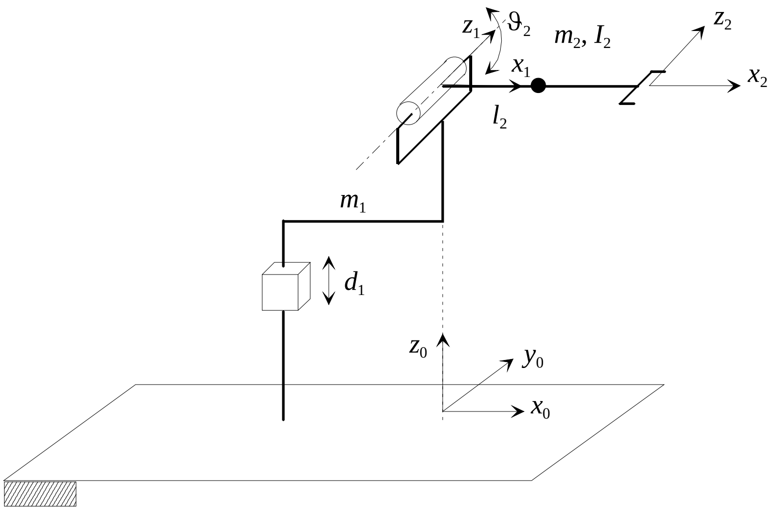
**Warnings:**

- This file consists of **8** pages (including cover). All the pages should be signed.
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

**Use this page ONLY in case of corrections or if the space reserved for some answers turned out to be insufficient**

**Exercise 1**

Consider the manipulator sketched in the picture:




**1.1** Find the expression of the inertia matrix of the manipulator.

Denavit-Hartenberg frames can be defined as sketched in the picture.

Computations of the Jacobians:

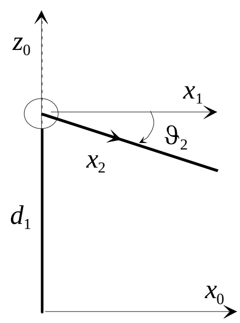
*Link 1*

$$\boldsymbol{J}_P^{(l_1)} = \begin{bmatrix} \boldsymbol{j}_{P1}^{(l_1)} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_0 & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

*Link 2*

$$\boldsymbol{J}_P^{(l_2)} = \begin{bmatrix} \boldsymbol{j}_{P1}^{(l_2)} & \boldsymbol{j}_{P2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{z}_0 & \boldsymbol{z}_1 \times (\boldsymbol{p}_{l2} - \boldsymbol{p}_1) \end{bmatrix} = \begin{bmatrix} 0 & -l_2 s_2 \\ 0 & 0 \\ 1 & -l_2 c_2 \end{bmatrix}$$

$$\boldsymbol{J}_O^{(l_2)} = \begin{bmatrix} \boldsymbol{j}_{O1}^{(l_1)} & \boldsymbol{j}_{O2}^{(l_2)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{z}_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$




Auxiliary vectors for the above computations:

$$\boldsymbol{p}_{l2} = \begin{bmatrix} l_2 c_2 \\ 0 \\ d_1 - l_2 s_2 \end{bmatrix}, \quad \boldsymbol{p}_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \quad \boldsymbol{z}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Inertia matrix:

$$\boldsymbol{B}(\boldsymbol{q}) = m_1 \boldsymbol{J}_P^{(l_1)T} \boldsymbol{J}_P^{(l_1)} + m_2 \boldsymbol{J}_P^{(l_2)T} \boldsymbol{J}_P^{(l_2)} + I_2 \boldsymbol{J}_O^{(l_2)T} \boldsymbol{J}_O^{(l_2)} = m_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + m_2 \begin{bmatrix} 1 & -l_2 c_2 \\ -l_2 c_2 & l_2^2 \end{bmatrix} + I_2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} m_1 + m_2 & -m_2 l_2 c_2 \\ -m_2 l_2 c_2 & I_2 + m_2 l_2^2 \end{bmatrix}$$

**1.2** Write the expression of the kinetic energy for this manipulator

$$T = \frac{1}{2} \dot{\boldsymbol{q}}^T \boldsymbol{B}(\boldsymbol{q}) \dot{\boldsymbol{q}} = \frac{1}{2} \begin{bmatrix} \dot{d}_1 & \dot{\vartheta}_2 \end{bmatrix} \begin{bmatrix} m_1 + m_2 & -m_2 l_2 c_2 \\ -m_2 l_2 c_2 & I_2 + m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{\vartheta}_2 \end{bmatrix}$$

**1.3** One of the properties of the dynamic model of the manipulator is skew-symmetry of a certain matrix. State what the skew symmetric matrix is and cite one problem where this property is used.

The following matrix:

$$\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \dot{\boldsymbol{B}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$$

is skew-symmetric, provided that the matrix $\boldsymbol{C}$ of the Coriolis and centrifugal terms is built based on the Christoffel symbols.

This property is used in the proof of stability of some centralized control schemes, like PD + gravity compensation.

**1.4** Explain how the Newton-Euler method can be used in order to compute the *direct dynamics* of a manipulator.

Assume that we have a Newton-Euler script in the form:

$$\boldsymbol{\tau} = \mathrm{NE}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$$

With the current values of $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$, a first iteration of the script is performed, setting $\ddot{\boldsymbol{q}} = 0$. In this way the torques $\boldsymbol{\tau}$ computed by the method directly return the vector of the Coriolis, centrifugal, and gravitational terms.

Then we set $\boldsymbol{g}_0 = 0$ inside the script (in order to eliminate the gravitational effects) and $\dot{\boldsymbol{q}} = 0$ (in order to eliminate Coriolis and centrifugal effects). $n$ iterations of the script are performed, with $\ddot{q}_i = 1$ and $\ddot{q}_j = 0$, $j \neq i$. This way matrix $\boldsymbol{B}$ is formed column by column and all elements to compute the direct dynamics are available.

**Exercise 2**

Consider the generation of a position trajectory in the Cartesian space. Select as an initial point $\boldsymbol{p}_i = [1, 2, 0]$ and as a final point $\boldsymbol{p}_f = [1, -1, 4]$.

**2.1** Write the expression of a segment connecting the initial and the final points, parameterized with the natural coordinate.

The general expression of the segment is:

$$\boldsymbol{p}(s) = \boldsymbol{p}_i + \frac{s}{\left\| \boldsymbol{p}_f - \boldsymbol{p}_i \right\|} (\boldsymbol{p}_f - \boldsymbol{p}_i)$$

Since:

$$\boldsymbol{p}_f - \boldsymbol{p}_i = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}, \quad \left\| \boldsymbol{p}_f - \boldsymbol{p}_i \right\| = 5$$

it is:

$$\boldsymbol{p}(s)=\begin{bmatrix}1\\2\\0\end{bmatrix}+\frac{s}{5}\begin{bmatrix}0\\-3\\4\end{bmatrix}$$

**2.2** Assume a travel time $T$ = 2 *s*. Design a trajectory, which covers the path determined in the previous step, using a cubic dependence on time.

We need to find the coefficients of the polynomial:

$$s(t)=a_0+a_1t+a_2t^2+a_3t^3$$

with the boundary conditions:

$$\begin{matrix}s(0)=0 & s(2)=5\\ \dot{s}(0)=0 & \dot{s}(2)=0\end{matrix}$$

From the conditions at $t$ = 0, we easily obtain $a_0$ = $a_1$ = 0. From the conditions at $t$ = 2 we obtain the linear system:

$$\begin{cases}4a_2+8a_3=5\\4a_2+12a_3=0\end{cases}\Rightarrow\begin{cases}a_2=15/4\\a_3=-5/4\end{cases}$$

Then the expression of the cubic polynomial is:

$$s(t)=\frac{15t^2-5t^3}{4}$$

**2.3** Compute the maximum linear velocity of the end effector along the trajectory designed in the previous step. If this maximum value exceeds the maximum achievable velocity how would you modify the trajectory generation?

The maximum linear velocity corresponds to the maximum value of the derivative of function $s(t)$:

$$\|\dot{\boldsymbol{p}}\|_{\max}=\dot{s}_{\max}=\dot{s}(1)=\frac{30-15}{4}=\frac{15}{4}=3.75$$

In case this value is too high, we need to rescale the trajectory, appropriately increasing the travel time.

**2.4** Suppose that the manipulator is kinematically redundant for the task of end effector positioning. Write the expression of the inverse kinematics based on the pseudo-inverse matrix and explain what is the optimization problem solved with this approach.

The expression of the inverse kinematics is:

$$\dot{\boldsymbol{q}}=\boldsymbol{J}^{\#}\dot{\boldsymbol{p}}$$

where $\boldsymbol{J}^{\#}$ is the pseudo-inverse of the Jacobian, defined as:

$$\boldsymbol{J}^{\#}=\boldsymbol{J}^T\left(\boldsymbol{J}\boldsymbol{J}^T\right)^{-1}$$

This solution solves an optimization problem, where the cost function to be minimized is the square norm of the joint velocities:

$$g(\dot{\boldsymbol{q}})=\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{\boldsymbol{q}}=\frac{1}{2}\|\dot{\boldsymbol{q}}\|^2$$

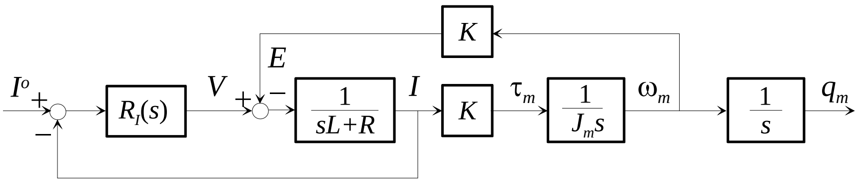
subject to the constraint:

$\dot{\boldsymbol{p}} - \boldsymbol{J}\dot{\boldsymbol{q}} = 0$

**Exercise 3**

Consider the control of a servomechanism.

**3.1** Sketch the block diagram of a current control (make reference to a common DC motor).




**3.2** Explain what are the advantages in using a closed loop current control.

Since the electrical dynamics is quite fast (compared with the mechanical one) we can design the current controller $R_I(s)$ so as to achieve a wide bandwidth (thousands rad/s). In the design of the current controller the e.m.f. can be considered as a slowly varying disturbance, that the controller can effectively reject. Once the current control loop is closed, it can be seen as practically instantaneous from the external position/speed controller, so that:

$$\tau_m(t) = KI(t) \approx KI^o(t)$$

This is why in the design of the speed controller for a servomechanism the electrical dynamics and the electrical parameters of the model can be neglected.

**3.3** Sketch a common Bode diagram for the design of the PI speed controller in a rigid servomechanism and derive the expressions for tuning the gain and the integral time of the controller.

The transfer function from torque to speed in a rigid servomechanism, neglecting friction, is:

$$G_v(s) = \frac{\mu}{s}$$

with:

$$\mu = \frac{1}{J_m + J_{lr}}$$

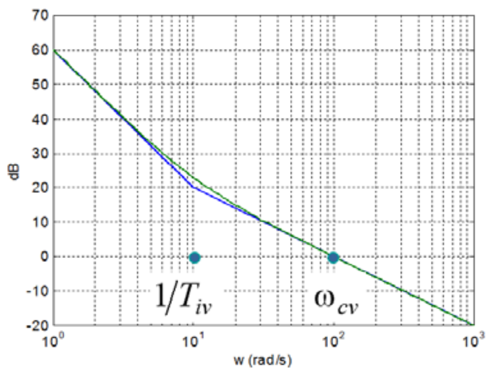
The transfer function of the PI controller is:

$$R_{PI}(s) = K_{pv}\left(1 + \frac{1}{sT_{iv}}\right) = K_{pv}\frac{1 + sT_{iv}}{sT_{iv}}$$

and then the loop transfer function is:

$$L_v(s) = R_{PI}(s)G_v(s) = \frac{K_{pv}\mu}{s}\frac{1 + sT_{iv}}{sT_{iv}}$$

A qualitative Bode diagram is here reported.




The crossover frequency $\omega_{cv}$ can be estimated with the high-frequency approximation of $L_v$, which yields:

$$\omega_{cv} = K_{pv}\mu$$

The zero of the controller can be placed in a sufficiently lower frequency range with respect to $\omega_{cv}$:

$$\frac{1}{T_{iv}} = (0.1 \div 0.3)\omega_{cv}$$

**3.4** Assume now the following values for the physical parameters:

$J_l$ = 5 Kg m$^2$, $\rho$ = 2, $n$ = 50.

Tune a PI speed controller for the servomechanism, for a crossover frequency $\omega_{cv}$ = 150 rad/s.

The load inertia reflected at the motor axis is:

$$J_{lr} = \frac{J_l}{n^2} = 0.002$$

The motor inertia is:

$$J_m = \frac{J_{lr}}{\rho} = 0.001$$

Then:

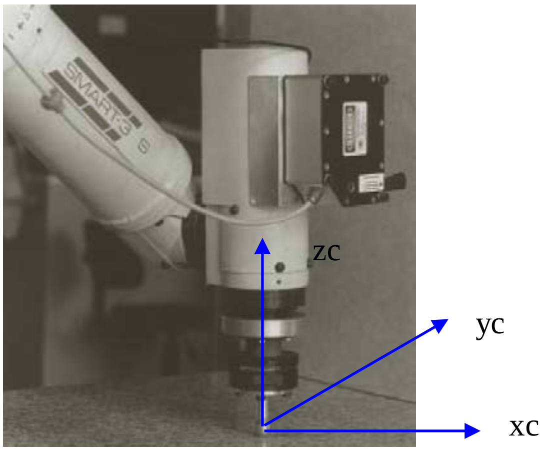
$$K_{pv} = \frac{\omega_{cv}}{\mu} = (J_m + J_{lr})\omega_{cv} = 0.003 * 150 = 0.45$$

The integral time of the controller can be set as:

$$T_{iv} = \frac{10}{\omega_{cv}} = \frac{10}{150} = 0.066$$

**Exercise 4**

Consider an interaction task of a manipulator, with a frictionless and rigid surface, as in this picture:




**4.1** Assume a point contact and draw a contact frame directly on the picture. Based on this frame and neglecting angular velocities and moments, express the natural and the artificial constraints for this problem.

| Natural Constraints | Artificial Constraints |
|---|---|
| $f_x^c$ | $\dot{p}_x^c$ |
| $f_y^c$ | $\dot{p}_y^c$ |
| $\dot{p}_z^c$ | $f_z^c$ |

**4.2** Write the expression of the selection matrix for this problem, explaining the meaning of such matrix.

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & 0 \\ 0 & 0 & \mathbf{1} \end{bmatrix}$$

The selection matrix extracts directions where a natural constraint in velocity applies and thus an artificial constraint in force can be enforced.

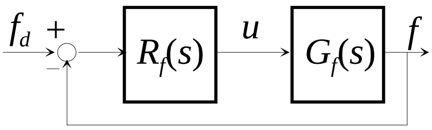
**4.3** Suppose that this interaction matrix has to be used in a hybrid force/position control scheme. Identify some possible sources of inconsistency in the measurements.

Hybrid position/force control is based on a nominal model of the interaction. Inconsistency may however occur in the measurements, due e.g. to:

1. friction at the contact (a force is detected in a nominally free direction)
2. compliance in the robot structure and/or at the contact (a displacement is detected in a direction which is nominally constrained in motion)
3. uncertainty in the environment geometry at the contact

**4.4** Suppose now that along the force controlled direction an explicit force controller has to be designed. Determine the expression of such controller, taking a bandwidth of 30 rad/s.

We can make reference to the following block diagram:




where $G_f(s)$ is a unitary gain transfer function with some high frequency resonance.

It is convenient to use an integral controller, with gain equal to the desired crossover frequency:

$$R_f(s) = \frac{K_{if}}{s} = \frac{30}{s}$$