POLITECNICO
MILANO 1863

Motion planning for mechanical systems

Academic year 2025-2026

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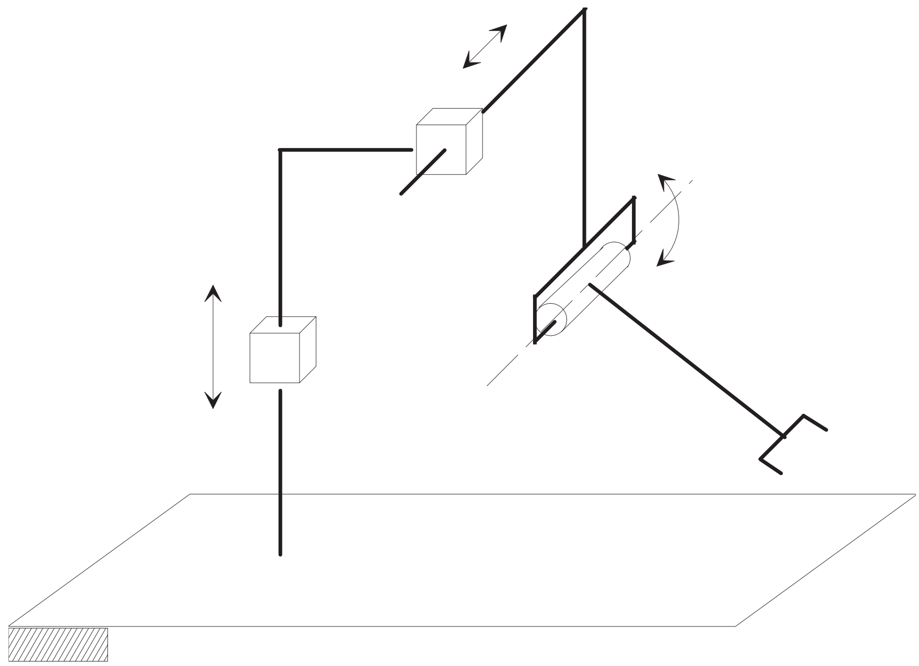
Warnings:

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Exercise 1

Consider the manipulator drawn in the figure:



Point 1.1 Plot, on the figure itself, the frames according to the Denavit-Hartenberg convention and fill in the relative table of parameters:

	a	α	d	ϑ
1				
2				
3				

Point 1.2 For the given manipulator, write the equations of the direct kinematics relative to position only. [1]

[1]Remember, if you consider it useful for the solution of the exercise, the expression of the homogeneous transformation matrix between two consecutive triplets:

$$\mathbf{A}_i^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i} c_{\alpha_i} & s_{\vartheta_i} s_{\alpha_i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i} c_{\alpha_i} & -c_{\vartheta_i} s_{\alpha_i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

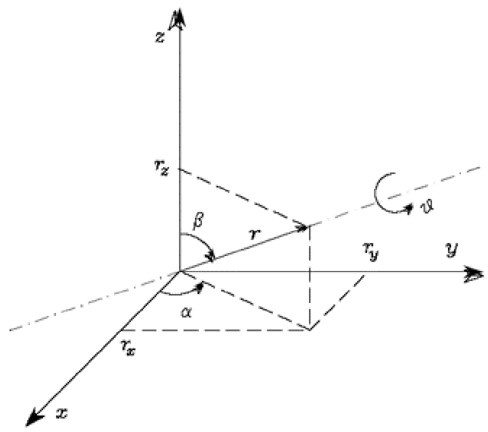
Point 1.3 For the given manipulator, determine the geometric Jacobian (relative to linear velocities only), highlighting the singularity points[2]

Point 1.4 For a generic manipulator, explain why kinematic singularities are problematic. How many singularities does a 6-dof anthropomorphic manipulator typically have?

[2]Remember that the vector product between the vectors $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is $c = a \times b = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$

Exercise 2

Point 2.1 Referring to the following figure, discuss the axis-angle representation of the orientation. In particular, explain how many parameters this representation is composed of and what relationship, if any, exists between these parameters.



Point 2.2 Now suppose you want to plan the orientation trajectory of a robot's end-effector. The rotation matrices that express respectively the initial orientation and the final orientation with respect to the world frame are:

$$\mathbf{R}_i = \mathbf{I}_3, \qquad \mathbf{R}_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Determine the axis around which the rotation is to be made and the angle by which it is to be rotated.

Point 2.3 With reference to the data of this exercise, plan the evolution of the angle with a cubic time law, with positioning time equal to $T = 1s$. Assume the initial and final velocities to be null and express the angle in degrees.[3]

Point 2.4 With the cubic profile previously used, are the angle, the related velocity, and the acceleration, continuous in all the time instants, including the initial and the final ones? Comment your answer.

[3] If you have not been able to solve point 2 of the exercise, solve this point by assuming the final value of the angle $40°$.

Exercise 3

Point 3.1 For a single rigid body, write the expression of the kinetic energy, specifying the meaning of all the symbols used.

Point 3.2 For a generic manipulator, write the expression of the kinetic energy. What are the properties of the inertia matrix?

Point 3.3 In the derivation of the Euler-Lagrange model of a systems of bodies, the Lagrange's equations are used. Write the general expression of such equations, defining in particular the concept of Lagrangian.

Point 3.4 Write the complete dynamic model of the robot, as derived from the Euler-Lagrange method. Specify the meaning of the symbols used.