POLITECNICO
MILANO 1863

Motion planning for mechanical systems

Academic year 2025-2026

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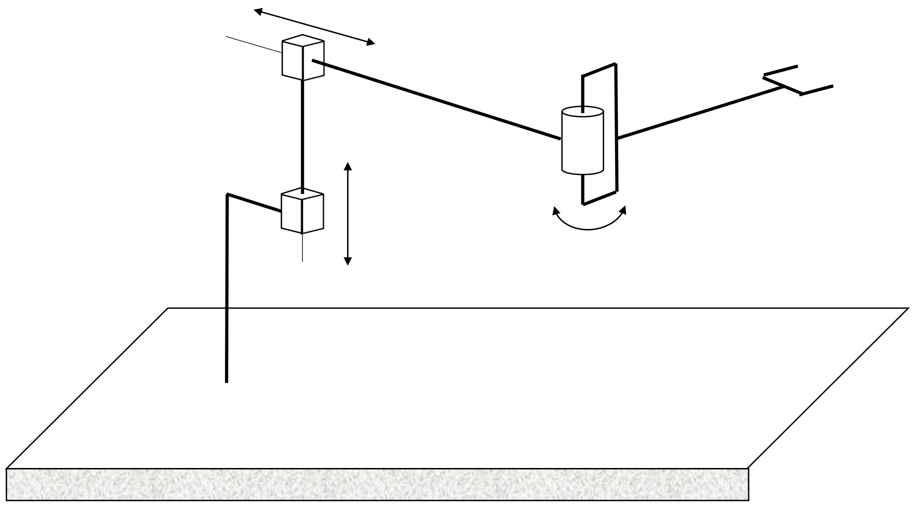
Warnings:

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Exercise 1

Consider the manipulator drawn in the figure:



Point 1.1 Plot, on the figure itself, the frames according to the Denavit-Hartenberg convention and fill in the relative table of parameters:

	a	α	d	ϑ
1				
2				
3				

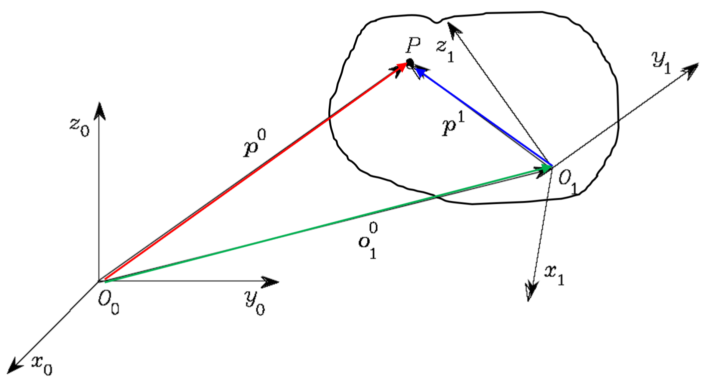
Point 1.2 For the given manipulator, write the equations of the direct kinematics relative to position only. [1]

[1] Remember, if you consider it useful for the solution of the exercise, the expression of the homogeneous transformation matrix between two consecutive frames is:

$$\mathbf{A}_i^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i} c_{\alpha_i} & s_{\vartheta_i} s_{\alpha_i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i} c_{\alpha_i} & -c_{\vartheta_i} s_{\alpha_i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Point 1.3 For the given manipulator, determine the geometric Jacobian (relative only to linear velocities), highlighting the singularity points

Point 1.4 Making reference to the following picture, write the expression of vector $\mathbf{p}^0$ in terms of vector $\mathbf{p}^1$.



Exercise 2

Point 2.1 Consider a trapezoidal velocity profile trajectory. Derive the velocity expression $\dot{q}_v$ in the middle section (cruising velocity) given the positioning time T, the acceleration time T_a and the distance to travel h.

Point 2.2 Now consider generating the trajectory with trapezoidal velocity profile for a variable $q(t)$ with the following data:

$$\begin{array}{rr} q_i = 0 & q_f = 40^\circ \\ \dot{q}_{\max} = 10^\circ/s & \ddot{q}_{\max} = 10^\circ/s^2 \end{array}$$

Determine the minimum positioning time T and its acceleration time T_a

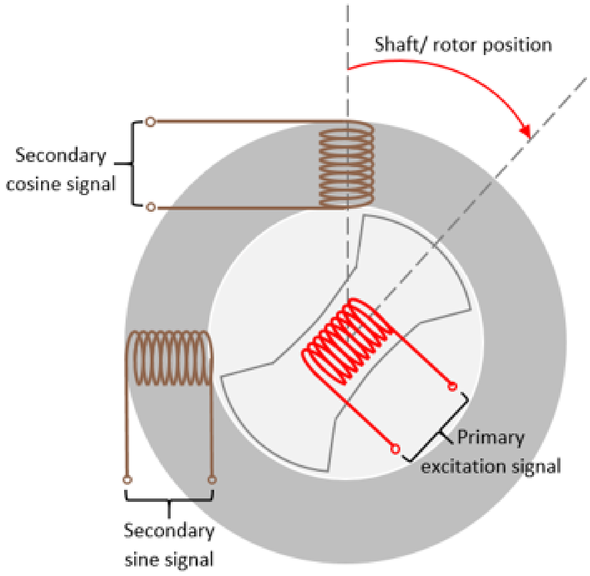
Point 2.3 With reference to the planning in the previous point, suppose that the positioning time has increased by 20%, without changing the acceleration time. Determine the cruising velocity that is obtained as a result of the change in the positioning time.

Point 2.4 Consider now the planning of a position trajectory in the operational space. Explain how a time law like the one elaborated in the previous points can be exploited in such planning.

Exercise 3

Point 3.1 Explain why in robotics dedicated systems to measure the joint positions are used.

Point 3.2 Referring to the following figure, briefly explain (without resorting to formulas) the operating principle of a resolver



Point 3.3 Now consider an absolute encoder. On the basis of the operating principle, derive the formula that expresses the resolution of the instrument. Then determine the minimum number of tracks to have a resolution of less than a tenth of a degree.

Point 3.4 An indirect measurement of speeds can also be obtained from encoder measurements. Explain the fixed-time and fixed-space methods used for this purpose.