POLITECNICO
MILANO 1863

Motion planning for mechanical systems

Academic year 2025-2026

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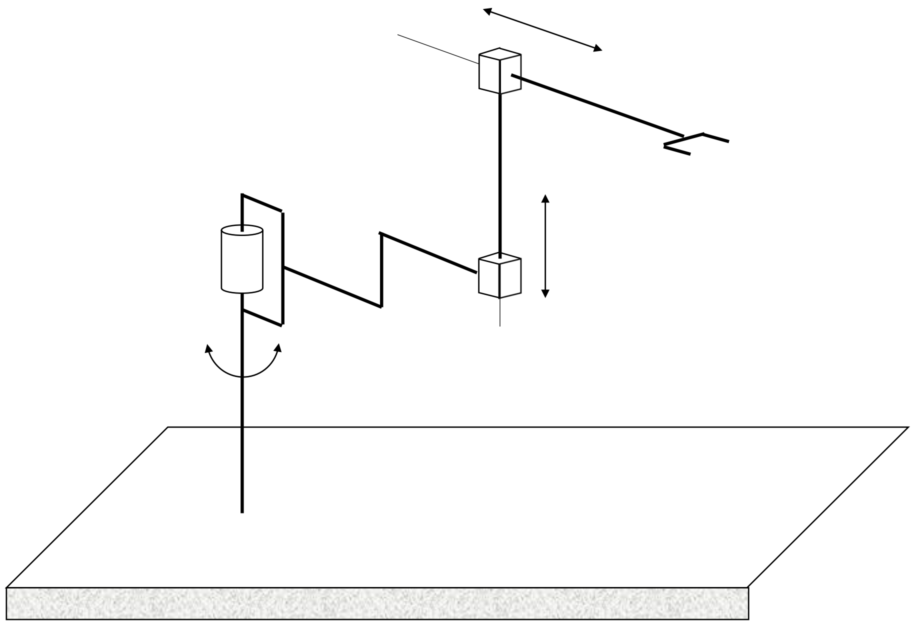
Warnings:

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

Exercise 1

Consider the manipulator drawn in the figure:



Point 1.1 Plot, on the figure itself, the frames according to the Denavit-Hartenberg convention and fill in the relative table of parameters:

	a	α	d	ϑ
1				
2				
3				

Point 1.2 For the given manipulator, write the equations of the directed kinematics relative to position only. [1]

[1]Remember, if you consider it useful for the solution of the exercise, the expression of the homogeneous transformation matrix between two consecutive frames:

$$\mathbf{A}_i^{i-1} = \begin{bmatrix} c_{\vartheta_i} & -s_{\vartheta_i} c_{\alpha_i} & s_{\vartheta_i} s_{\alpha_i} & a_i c_{\vartheta_i} \\ s_{\vartheta_i} & c_{\vartheta_i} c_{\alpha_i} & -c_{\vartheta_i} s_{\alpha_i} & a_i s_{\vartheta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Point 1.3 For the given manipulator, determine the geometric Jacobian (relative only to linear velocities), highlighting the singularity points

Point 1.4 For the manipulator of this exercise in a generic configuration, write the expression of the homogeneous transformation matrix of the frame 3 with respect to the frame 0.

Exercise 2

Point 2.1 Consider a trapezoidal velocity profile trajectory. Suppose that the positioning time T and the distance to travel h are assigned. Given a value of the velocity $\dot{q}_v$ in the middle section (cruising speed), how can the acceleration time T_a and the initial acceleration $\ddot{q}_c$ be determined? What condition must the assigned cruising speed verify for the problem to be solved?

Point 2.2 Now consider generating the trajectory with trapezoidal velocity profile for a variable $q(t)$ with the following data:

$$\begin{array}{rr} q_i = 0 & q_f = 20^\circ \\ \dot{q}_{\max} = 10^\circ/s & \ddot{q}_{\max} = 10^\circ/s^2 \end{array}$$

Determine the minimum positioning time T and its acceleration time T_a

Point 2.3 For the same positioning discussed in point 2.2, assuming zero initial and final velocity, determine the expression of a cubic trajectory.

Point 2.4 Compute the maximum speed for the cubic trajectory and compare such value with the cruise speed of the trapezoidal velocity profile previously discussed.

Exercise 3

Point 3.1 Consider a rigid body, for which the orientation with respect to a reference frame has to be expressed. Explain what are the ZYZ Euler angles.

Point 3.2 Define the angular velocity vector for a rigid body. Is the angular velocity vector the derivative of a set of Euler angles?

Point 3.3 Define the linear momentum and the angular momentum of a rigid body.

Point 3.4 The dynamics of a rigid body can be written in terms of the Newton-Euler equations: write such equations for a generic rigid body.