Warnings

- This file consists of 8 pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given either in English or in Italian.
- Solutions and answers must be given exclusively in the reserved space. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to hand this file only. Every other sheet you may hand will not be taken into consideration.
EXERCISE 1

1. With reference to a generic control system

\[ y^o + e \rightarrow L(s) \rightarrow d + y \]

Give the definition of sensitivity function of the system, explaining its use.

The sensitivity function is defined as:

\[ S(s) = \frac{1}{1 + L(s)} \]

It is the transfer function from a disturbance to the output in a closed loop control system.

2. Suppose that the loop transfer function \( L \) has the Bode plot of the magnitude as shown in the figure:

Sketch the asymptotic Bode plot of the magnitude of the sensitivity function.

We can rely on the approximation:

\[ |S(j\omega)| = \frac{1}{|L(j\omega)|} \approx \frac{1}{|L(j\omega)|} \quad \text{for} \quad L(j\omega) \gg 1 \]

\[ |S(j\omega)| \approx 1 \quad \text{for} \quad L(j\omega) \ll 1 \]
3. Consider a disturbance \( d(t) = \sin(0.3t) \). Compute the factor by which the disturbance is attenuated.

For \( \omega = 0.3 \text{ rad} \), the amplitude of \( S' \) is -40 dB, which corresponds to 0.01.

Therefore, the attenuation factor is 100 (the effect of the disturbance on the output is attenuated by a factor 100).
EXERCISE 2

1. Explain what is intended with logic control and what is the role of a PLC in an automation system.

The logic control is the control of sequences of actions that need to be executed when triggered by events and in the correct order.

A Programmable Logic Controller is a device that receives as an input some logic variables and can be programmed to output other logic variables.

2. Consider now an automatic irrigation system. Pressing a button START the automatic irrigation system is activated for one hour. After such time interval, the pump of the irrigation system turns off and for 24 hours pressing the button START cannot activate the irrigation system. Program the system with a Ladder Diagram code.

We can use as an input variable START, as an output a variable IRMG, two timers, and also an internal variable to give consensus to the irrigation.

Pressing start latches the variable CONS

As CONS is true, the first timer counts and the irrigation system is active.

The first timer triggers the second one

The second time unlatches the CONS variable.
3. Explain what is the meaning of the following symbol in a Ladder Diagram code:

```
--|P|--
```

This symbol is a positive edge detection contact. It closes for a single cycle when the associated variable changes from 0 to 1.

4. Consider now a digital network with bus architecture: describe two possible ways to handle the access to the bus among the various agents.

**CSMA/CD** (the one used by Ethernet):
- any node can attempt to transmit on the channel,
- collisions can occur and then the sender can try again

**Token bus**
- only the node that has a special code (token) can use the channel
- the token is passed among the nodes
EXERCISE 3

1. Consider the following robot manipulator with 3 joints (rotational, prismatic and prismatic):

Find the expression of the direct kinematics of the robot, in terms of the position coordinates of the end effector with respect to the joint variables \( \theta_1, d_2, \) and \( d_3 \).

We can represent the robot with a projection on plane \((x_0, y_0)\)

Clearly:
\[
\begin{align*}
P_x &= d_2 \cos(\theta_1) \\
P_y &= d_2 \sin(\theta_1)
\end{align*}
\]

By looking again at the 3D sketch:
\[
P_z = d_1 + d_3
\]
2. Write the expression of the Jacobian of this manipulator.

The Jacobian takes this expression:

\[
J = \begin{bmatrix}
\frac{\partial P_x}{\partial \theta_1} & \frac{\partial P_x}{\partial \theta_2} & \frac{\partial P_x}{\partial \theta_3} \\
\frac{\partial P_y}{\partial \theta_1} & \frac{\partial P_y}{\partial \theta_2} & \frac{\partial P_y}{\partial \theta_3} \\
\frac{\partial P_z}{\partial \theta_1} & \frac{\partial P_z}{\partial \theta_2} & \frac{\partial P_z}{\partial \theta_3}
\end{bmatrix} = \begin{bmatrix}
-d_2 \sin(\theta_2) & \cos(\theta_2) & 0 \\
-d_2 \cos(\theta_2) & \sin(\theta_2) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

3. Characterize the singularities of this manipulator.

The determinant of the Jacobian is:

\[
\det(J) = -d_2 \sin(\theta_2) \cdot d_2 \cos(\theta_2) = -d_2^2
\]

It is equal to zero if and only if \(d_2 = 0\).

Therefore, the only singularity is when \(d_2 = 0\), i.e., the end effector is on the axis of the first joint.