Industrial Automation and Robotics

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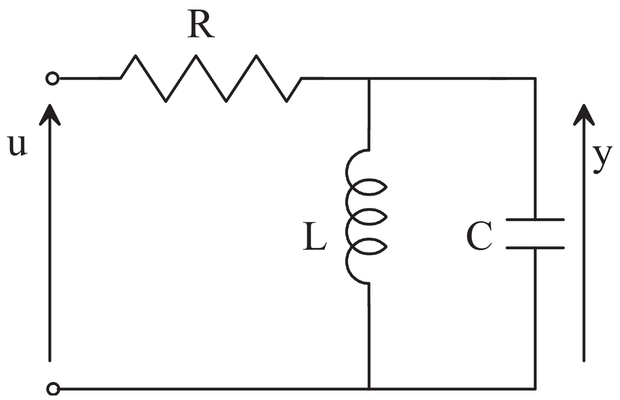
Warnings

- This file consists of **8** pages (including cover).
- During the exam you are not allowed to exit the room for any other reason than handing your work or withdrawing from the exam.
- You are not allowed to withdraw from the exam during the first 30 minutes.
- During the exam you are not allowed to consult books or any kind of notes.
- You are not allowed to use calculators with graphic display.
- Solutions and answers can be given **either in English or in Italian**.
- Solutions and answers must be given **exclusively in the reserved space**. Only in the case of corrections, or if the space is not sufficient, use the back of the front cover.
- The clarity and the order of the answers will be considered in the evaluation.
- At the end of the test you have to **hand this file only**. Every other sheet you may hand will not be taken into consideration.

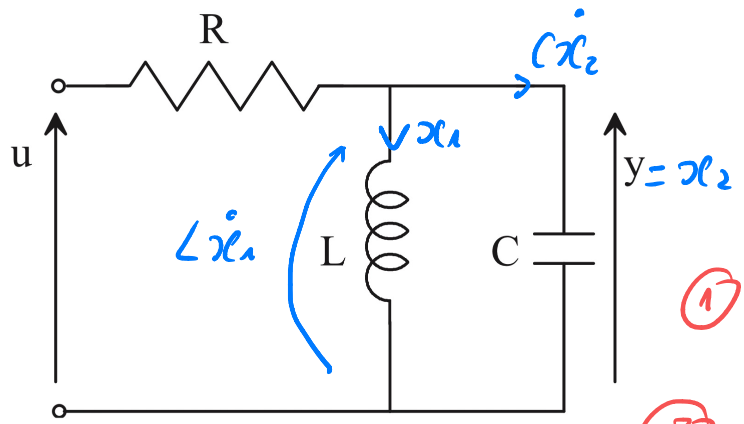
EXERCISE 1

1. Consider the electrical network sketched in the figure:



Write the equations of the dynamic system that describes the electrical network.

The dynamic system is a second order one. We can select as state variables the current in the inductor and the voltage across the capacitor.



We can write:

$$\begin{cases} L\dot{x}_1 = x_2 & (1) \\ u = x_2 + R(x_1 + C\dot{x}_2) & (2) \end{cases}$$

(1) Voltage on the inductor = voltage on the capacitor

(2) u = voltage on capacitor + voltage on resistor

$$\begin{cases} \dot{x}_1 = \frac{1}{L} x_2 \\ \dot{x}_2 = -\frac{1}{C} x_1 - \frac{1}{RC} x_2 + \frac{1}{RC} u \end{cases}$$

2. Find the equilibrium state and the equilibrium output corresponding to a constant input $u = \bar{u} = 2$

To find the equilibrium state we need to set the derivatives to zero

$$\begin{cases} \frac{1}{L}\bar{x}_2 = 0 \\ -\frac{1}{C}\bar{x}_1 - \frac{1}{RC}\bar{x}_2 + \frac{1}{RC}\bar{u} = 0 \end{cases} \Rightarrow \begin{cases} \bar{x}_1 = \frac{\bar{u}}{R} = \frac{2}{R} \\ \bar{x}_2 = 0 \end{cases}$$

3. Find the expression of the transfer function from the voltage input u to the voltage output y

To find the transfer function, we replace the derivative with the operator s:

$$\begin{cases} sX_1 = \frac{1}{L} X_2 \\ sX_2 = -\frac{1}{C} X_1 - \frac{1}{RC} X_2 + \frac{1}{RC} U \end{cases}$$

$$Y = X_2$$

We eliminate X_1: $X_1 = \frac{1}{Ls} X_2$

$$sX_2 = -\frac{1}{LCs} X_2 - \frac{1}{RC} X_2 + \frac{1}{RC} U \Rightarrow X_2 = \frac{Ls}{RLCs^2 + Ls + R} U$$

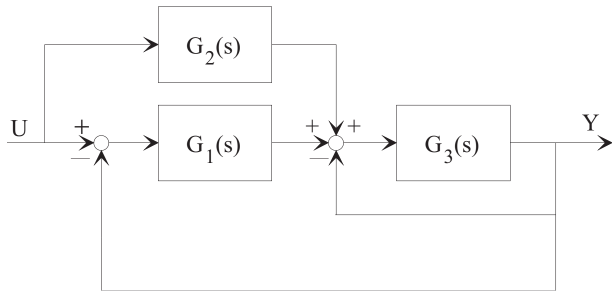
Transfer function $G(s) = \frac{Y(s)}{U(s)} = \frac{Ls}{RLCs^2 + Ls + R}$

4. What is the type of the transfer function determined in the previous step?

Since the transfer function has a zero in $s=0$, the type is $g=-1$.

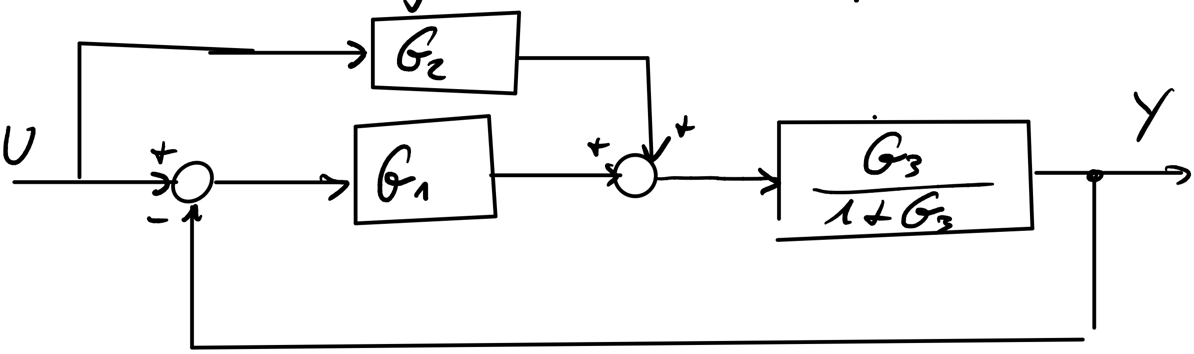
EXERCISE 2

1. Consider the dynamical system described by the following block diagram:



Solve the block diagram by determining the transfer function from u to y.

The block $G_3(s)$ is closed in a unitary feedback loop.
We can then simplify the block diagram as:



We can now use directly the formula for closed loop system:

$$\frac{Y}{U} = \frac{\text{t.f. of the forward path}}{1 + \text{t.f. of the loop}}$$

The foward path is G_1 in parallel with G_2 and the result in series with $G_3/1+G_3$, the loop by G_1 in series with $G_3/1+G_3$:

$$\frac{Y}{U} = \frac{(G_1+G_2)\frac{G_3}{1+G_3}}{1+G_1\cdot\frac{G_3}{1+G_3}} = \frac{(G_1+G_2)G_3}{1+G_3+G_1\cdot G_3}$$

2. Discuss whether it is necessary and/or sufficient that one or more of the transfer functions be asymptotically stable in order for the overall system to be asymptotically stable

Since $G_2(s)$ is not closed in a feedback loop, it is necessary that $G_2(s)$ is asymptotically stable for the overall system to be.

3. Setting $G_1(s) = 1$, $G_2(s) = 0$, $G_3(s) = \frac{1}{s}$ discuss the stability of the overall system

Substituting:

$$\frac{Y}{U} = \frac{\frac{1}{s}}{1 + \frac{1}{s} + \frac{1}{s}} = \frac{1}{s+2}$$

This transfer function has a single pole in $s = -2$, therefore it is asymptotically stable.

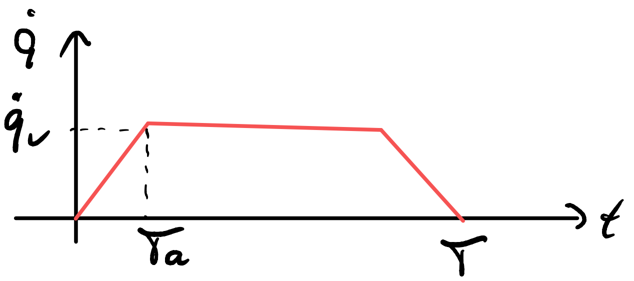
EXERCISE 3

1. Explain what is the difference between the trajectory generation for a robot in the joint space and in the operational space.

Joint space: each joint variable evolves from its initial to its final value independently. No kinematic inversion, no issues with singularities, motion of end effector unpredictable.

Operational space: the position and orientation of the end effector evolve along a path. Full control of the e.e. but issues with singularities.

2. Consider now the design of a joint trajectory with a trapezoidal velocity profile. The total displacement is $h = 20$, the total positioning time is $T = 1s$ and the acceleration time is $T_a = 0.25s$. Compute the constant speed in the central part of the motion.

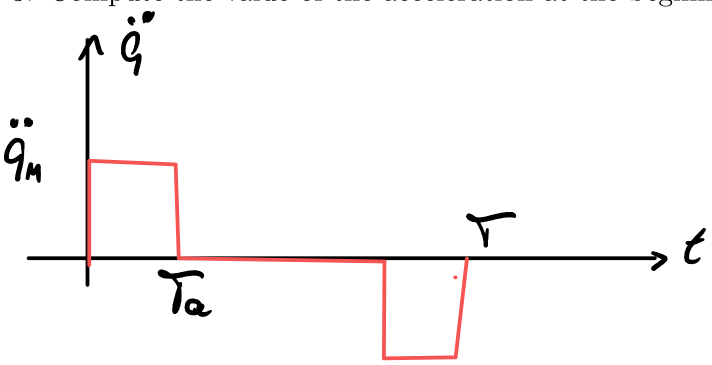


The total displacement is the area of the velocity trapezoidal:

$$h = 2 \cdot \frac{1}{2} \cdot \dot{q}_v T_a + \dot{q}_v (T - 2T_a) = \dot{q}_v (T - T_a)$$

Therefore $$\dot{q}_v = \frac{h}{T - T_a} = \frac{20}{1 - 0.25} = 26.66$$

3. Compute the value of the acceleration at the beginning of the trajectory



Clearly $\dot{q}_V = \ddot{q}_M \cdot T_a \implies \ddot{q}_M = \dfrac{\dot{q}_V}{T_a} = 106.66$

4. Suppose now that the maximum available acceleration is the same found at the previous step, while the maximum available speed is one half of the speed computed previously. For the same displacement of this exercise, find the minimum positioning time.

When we can use both the maximum speed and the maximum acceleration, we can write:

$h = \dot{q}_{max}(T - T_a)$

$\dot{q}_{max} = \ddot{q}_{max} T_a \implies T = \dfrac{h}{\dot{q}_{max}} + \dfrac{\dot{q}_{max}}{\ddot{q}_{max}}$

In this case, $\dot{q}_{max} = \dfrac{\dot{q}_V}{2} = 13.33 \qquad \ddot{q}_{max} = \ddot{q}_M = 106.66$

Then: $T_{min} = \dfrac{20}{13.33} + \dfrac{13.33}{106.66} = 1.62\ s$

(60% more than the original positioning time)